

THE
YOUNG ALGEBRAIST'S
COMPANION,
OR,
A NEW and EASY GUIDE to
ALGEBRA;

Introduced by the Doctrine of

VULGAR FRACTIONS:

Designed for the Use of SCHOOLS, and such
who, by their own Application only, would be-
come acquainted with the Rudiments of this noble
Science :

Illustrated with

Variety of numerical and literal Examples, and at-
tempted in natural and familiar Dialogue, in order to
render the Work more easy and diverting to those that
are quite unacquainted with *Fractions* and the *Analytic*
Art.

THE SECOND EDITION Corrected.

To which is added,

AN APPENDIX

On the Rudiments of Quadratic Equations, and a new and
easy geometrical Definition of the Difference between the solid Con-
tent of the Cylinder, and the Parallelopiped proved by the Pen.

By DANIEL FENNING, of the
ROYAL-EXCHANGE ASSURANCE.

L O N D O N :

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John S. West,
Tiverton, R. I.



DEDICATION.

*To the Honourable the GOVERNORS
and DIRECTORS of the ROYAL EX-
CHANGE ASSURANCE COMPANY.*

GENTLEMEN,



IT being not only a common, but a very just Observation, that the Generality of *Dedications* are carried to too high an Extreme of flattering Encomiums, I perswade myself you will the more readily accept of one in a *plain* Dress only, as I am sensible it is no-ways agreeable to your *Honours* to be flattered:

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When solicited by several Gentlemen in Town and Country to publish the following Tract, I was not long considering at whose Feet to lay it: The Presumption indeed was so great, that though the Thoughts of former Favours in some Measure encouraged me, yet it was with Reluctance I presumed to beg *this*; and your permitting me to send it Abroad under such Protection, makes it difficult to say which is greater, the Honour, or the Kindness you have done me, since from the *One* I shall naturally reap the Benefit of the *Other*.

Even Books of *common Arithmetic* only have seldom wanted Patronizers, but Gentlemen of the first Class have more especially condescended to protect and maintain the more superior Parts of *Mathematical* Learning, not only from the Pleasure and Satisfaction that arises even from the very *speculative* Knowledge of them, but by being so useful in exercising the Faculties, and bringing the Mind into a just and proper Method of Reasoning, were always thought worthy the Study of the best of Men, are esteemed as pretty Decorations

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tions and Accompanyments to other Branches of Learning, and for Want of which, Education is always counted Something the less complete.

It is superfluous indeed to mention this, since it is evident your *Honours* were sensible of the Truth of it, from your Readiness to promote a Work wrote by one who at best professes himself but a *No-vice* in the thorough Knowledge of this extensive Study: So that your Love for Arts and Sciences is the more conspicuously seen, and you will appear upon the List with those worthy Patriots, whose Generosity and Benevolence prompt them daily to support and encourage all Sorts of Literature.

As I am sensible your *Honours* expect Nothing from me but Thanks and Duty, the first I most humbly return; and as I am conscious I have hitherto been faithful and diligent in your Service, I hope I shall always endeavour, to the best of my Abilities, to continue so, in due Sense of the many Kindnesses I have received from you.

That

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That the same kind Providence that recommended me to your *Honours* Favour, may continue to each Member the Blessing of Health, and *that* of Prosperity to the Company in general, is, without Doubt, the hearty Desire of many; but of none more than,

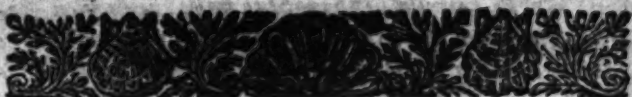
Gentlemen,

Your Honours most oblig'd,

and obedient,

humble Servant,

D. FENNING.



P R E F A C E.

Kind READER,

MANY and great are the Advantages that arise from the Knowledge of common Arithmetic only, to be Master of which requires a great deal of Time and Practice; but Algebra so far exceeds it, that were it not but that some sensible Progress must be made in the Former, it would hardly bear any Comparison with the Latter.

It may indeed be thought a needless and no less vain Attempt to offer any Thing of this Sort to the Public, that has already been treated of by so many eminent Authors, as if more could be said upon the Matter than has already been done. But it is to be observed that the following Work is designed only as a Preparative to the right understanding of other Authors; so that perhaps, upon due Consideration, the Undertaking may be found to be neither impertinent nor improper: For the Question is not, Are there not Books enow extant? but, Whether the Generality of them are fit for Learners, so that they may be capable in a short Time of themselves to solve Questions with the Help of a Master? This I think is the grand Point to be considered.

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I do not mean in the least to lessen those Authors, for whose Works I would be thought to have the highest Regard; but from the little Progress that most make who undertake to learn Algebra barely by Book, one would conclude (and that without the least Detraction) that the Rules, Instructions, and Examples, are not so plainly suited to the Capacities of Learners as could be wish'd.

A great many are very ambitious to purchase for their very first Book some great and noted Author (and indeed such must be consulted, if we would become Masters of any Art or Science;) but it is a common Mistake to think they are best, and most to be depended upon in the Rudiments or fundamental Principles of such Arts or Sciences. It is evident there is Something more in this than barely asserting it; for if it were so, what should be the Reason that so many inferior Authors (perhaps of ten Times less Knowledge in the Art or Science itself) have wrote, and do still continue to write Volumes, for the better understanding and explaining their Works?

If indeed there was no Occasion for this, then have they spent their Time in vain, and their Labours to no Purpose: But certainly they have not done so; it is what was wanting, and therefore a very necessary Undertaking; because by thus doing the Unlearned have and still may, become acquainted with such Mathematical Books, which otherwise they never would understand, without having a Tutor always at their Elbow; and then indeed Pardie's Geometry may be understood as well as Leadbetter's Mathematical Companion, or Boad's Artium Principia: And Wolfius and De Billy as well as Cocker or Hammond, but not without this or some such like Advantages. Besides,

It has long since been allow'd, that it is none of the easiest Things for Men of profound Learning to write within the ordinary Compass of common Capacities: Their Knowledge will very rarely suffer them to stoop to the
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the Understanding or Conception of such : So that what Dryden says of Love and Wisdom together, may in another Sense be applied to the Case before us :

“ The Proverb holds that to be wise and love,
“ Is hardly granted to the God’s above.”

For indeed, to Men of such extraordinary Parts and Abilities, every Thing of this Sort is so very easy, that whatever they propose, be it either Precept or Example, they can distinguish it so many different Ways, that from hence they conclude it is very easy to be understood by others ; on which Account they are in general not only so sublime, but withal so concise, that it is well known, that not only most of their easiest Problems (as they are pleased to call them) but even their Demonstrations, require a further Demonstration and more easy Explanation, before most Learners can of themselves form a clear and distinct Idea of the Nature of the Proposition ; at least it will cost them many a weary Hour, and sometimes Days and Months, to understand the whole Operation, though they apply ever so closely and assiduously ; and all this for Want of a few Words applied in a free, natural, and easy Manner. Indeed, when we consider these Things, we cannot but say it is Pity a diligent Reader should spend his Time to so little Purpose.

As Algebra is noted for its Excellency, so it is for its Difficulty, and therefore several eminent Authors have been pleased to call it a dark and dry Study ; the Meaning I apprehend to be this, because the Learner goes on a long Time through a Series of Rules and Examples, which, though he be ever so perfect in, yet sees no Reason for what he has done, nor receives any Relish or Satisfaction till he comes to put them in Practice in the solving of a few Questions ; and even then, if the Author does not give a Reason for almost every Step of the Operation,

Operation, the young Tyro will not so soon understand the Work as he may imagine, but is often at a Loss, and sometimes totally stopped.

Now if the young Algebraist may possibly by some Authors proceed thus far, and for Want of better Assistance can go no further, what can be thought of such Books as have not sufficient Instructions for the very Rudiments only: And that there are such is evidently known to too many, as will appear from the Words of an Author himself. — “ I have always been of Opinion, says he, that Algebra should not be entangled with a great Number of Precepts; the Science is dark enough, without adding to it new Obscurity by the Confusion of different Operations, &c.” — Had he said confused Operations indeed it had been to the Purpose; but what he means by calling different Operations a Confusion, I know not: For Reason (I think) tells us, that a Variety of Examples plainly demonstrated, is the most ready Way to remove Obscurity, as it is the only Means to prevent a Learner’s being entangled; because without Examples he would have but imperfect Ideas, and consequently could never understand the practical Part of any Art or Science: It is therefore not only disingenuous to own a Thing to be hard, and not witbal to give sufficient Rules and Examples to render it easy when in our Power; but it is as absurd to think it should ever be rightly understood without these.*

He is no great Arithmetician who will not allow the Doctrine of Fractions, and the Extraction of Roots to be something more difficult than the lower Rules of common Arithmetic; for which Reason the Directions are so many the more, and so much the fuller: If then

* It is of no Signification to mention the Author’s Name. Those that have him by them may see the same Words; and I make no Doubt but they will find he has kept up to them throughout his Work.

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to understand these only, such a Number of Cases and Examples are necessarily required, is it reasonable to suppose Algebra is to be learnt without, or by a few only? No surely; for look but into the Works of the unparalleled Saunderson, you will there find Precept upon Precept, and Example upon Example; and were it so that two Volumes in Quarto could be purchased by every Lover of this Science, they would (in all Probability) make more Algebraists than all the Books extant would for some Years put a Stop to any further Pretences, and may be said to be almost a finishing Stroke upon the Subject.—But this cannot be done.

Now, contrary to this, several Authors are so over and above short and intricate, that it is almost impossible for a Beginner to learn so much as the Algorithm, much less the Algorism*.

What may be done by one in a Thousand is not easily accounted for. Heaven has favoured some with a surprising quick Apprehension, a penetrating Judgment, and tenacious Memory; and if to these we add the Advantages of Time, together with the Delight and Operosity that such a one may possibly take in any Branch of Learning, such a Man as this, I say, cannot fail to make sensible Improvements from the most sublime Authors, and the most intricate Demonstrations: But can we expect to find this at every Door? Very few have two of all these Advantages, and therefore that which is but A, B, C to the one, is as Greek to the other.

From a due Consideration of these Things, it is easy to perceive that a Book wrote in a plain and familiar Manner, and with a moderate Price, has been long wished for and expected; the Want of which (as several

* Algorithm signifies the first Principle, and Algorism the practical Part, or knowing how to put the Algorithm in Practice.

veral Schoolmasters, and others my Acquaintance, have often said) has been a very great Discouragement to Learners in their intended Design and Pursuit of Algebra.

The most rational Method to make any one Master of any Art or Science, is certainly to introduce it to him in a natural Order, and to teach him at first only so much of it as is most necessary and consistent with the Foundation; and when he understands the Fundamentals, he will soon be able to conquer the more difficult Parts, and may pursue the Study in a more learned Manner, if his Fancy, Inclination, or Profession, shall incline him so to do.

Such then is the Design of the following Sheets, to give the Learner a true Notion of Simple Equations only; and to make it as useful to him as I possibly could, I have added that Part which treats of Vulgar Fractions, lest he should not be acquainted with so necessary a Step: And though I have laid down every Thing in the first four Algebraic Rules as plain as I am able, still, that the young Practitioner may not be at a Loss in working the Problems, I have there recalled him back to his former Work, have given him the Reason gradatim (Step by Step) throughout the whole Operation, that he may the more readily understand the Nature of what he has been doing.

I expect some critical Adepts may say, there was no Occasion to be so very particular; but let them be told once more, it is designed only for Learners, (though they themselves, perhaps, may find some Things in it not altogether unworthy their Notice;) and I persuade myself, upon this Consideration, I have not dwelt too long upon any one Thing that requires a clear Demonstration, if it were only for this Reason, that considering the many Rules and Examples the Learner has got to go through before he can put them in Practice,

and

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and the Difficulty of putting them in Practice to his own Satisfaction, an Author may be, and often is, too short for most, but he never can be too plain for any; and I think the practical Part of Algebra has already been sufficiently prov'd to be inconsistent with too much Sublimity and Conciseness. Again,

Does not Reason itself tell us, that Arts and Sciences are not like History. A few Words in the different Circumstances and Parts of the Narration give us a general Idea of the Whole: But here it is quite otherwise; the Reader must have Words as it were continually multiplied, to understand truly what is before him; so that that which may be called a needless Tautology in the one, is Nothing more than a proper and necessary Repetition or Addition to the other.

I hope, therefore; those more skilled in this Science will excuse my being a little prolix, if I tell them I have done it in Sympathy to the young Practitioner, since I know the dear Purchase of studying Algebra from concise and obscure Books by woful Experience!

It is true, I have attempted the Work in Dialogues, which render it more prolix than it would have been without them; but as I had never seen any Thing of this Sort wrote in that Manner, I did it partly to avoid the Charge of Plagiarism; and it is the general Opinion, that this Way of Writing conveys the Sense of the Matter sooner to the Ideas, as it unbends the Mind at Intervals, not by turning it aside from the Subject itself, but insensibly steals upon the Fancy, and renders the Study in a great Measure a Diversion rather than a dry burthensome Task. But a benevolent Critic knows,

“Whoever thinks a faultless Piece to see,

“Thinks what ne'er was, nor is, or e'er shall be.”

And a candid Judge, when he considers the Scope of the Science, and Design of the Author will grant,

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That

That if the Way be just, the Conduct true,
Some Praise, in Spight of trivial Faults, is due.

Let therefore the ingenuous Reader consider, that every Day has its Shades, and that my chief End is to serve him, and to save him Trouble in the Pursuit of this excellent Study, and then he will, for the very Design's Sake, forgive those Errors which the Press, Want of Time, or Ability, may possibly have occasioned.

Yours, &c.

Royal Exchange,
London, April
28, 1750.



To the AUTHOR of the YOUNG ALGEBRAIST'S COMPANION, &c.

S I R,

THERE is Nothing can give a greater Satisfaction to a Lover of the Sciences, than to see them handled in a clear and masterly Manner; and that every Attempt to remove Difficulties, and clear up the Obscurities of any knotty Part of Science (so as to level it to the Capacities of Youth, and at the same Time to make it pleasing as well as instructing) must be allow'd, by *all*, to be a difficult Task. Give me Leave therefore to thank you for the laudable Pains you have taken in your YOUNG ALGEBRAIST'S COMPANION, wherein you have made *that* which was hard and difficult, plain and easy to be understood, and at the same Time, have wrote it in so engaging a Manner, that the most inattentive Peruser must receive at once both Pleasure and Information. I am,

S I R,

Your unknown Friend

and humble Servant,

TOWER-ROYAL,
London, Aug. 21,
1750.

SAMUEL HILL.
Philom.

NB. *The Author has had several such friendly Letters as this, from other Strangers, in the City and Country.*



WE whose Names are here under-written, having each of us perused the following Sheets, do allow the Dialogues and Demonstrations to be very natural, and easily adapted; and therefore beg Leave to recommend the Work, as one of the plainest and best suited to the Capacities of young Beginners-extant.

ANTHONY GILBERT,
PETER DENNIS, Surveyor,
THOMAS HUMPHREYS,
TIMOTHY LANGLEY, Accomptant,
JOSEPH SIMPSON, Teacher of the Mathematics,
JOHN REPTON, Supervisor,
ABRAHAM DE LIRE, Teacher of the Mathematics,
JOHN QUANT, Writing-Master and Accomptant,
GEORGE COLES, Land-Surveyor,
RICHARD RICHARDSON,
ERASMUS TURNER,
ZACHARIAH SNAPER, Accomptant,
SAMUEL HILL, Philom.



INTRODUCTION.

CHAP. I. DIALOGUE I.

Between PHILOMATHES and TYRUNCULUS.

Tyr. ——— (at Philomathes's Door——knocks.)

Servant. ———

Tyr.  RAY is *Philomathes* within?

Serv. Yes, Sir.

Tyr. Has he Company?

Serv. No, Sir: Please to walk into the Parlour, my Master is quite alone.

Tyr. ——— (within) I choose rather you would let him know I am here.

Serv. ——— (to Philomathes) Sir, here's a Gentleman desires to speak with you, if you be at Leisure.

Phi. Who is it, *Psapho*?

Serv. I have seen the Gentleman before, Sir, but I forget his Name.

Phi. ——— (comes.)

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Tyr.

INTRODUCTION.

Tyr. Dear *Philomathes*, I am your humble Servant.

Phi. *Tyrunculus*, I am yours, and am heartily glad to see you. — Come, Sir, pray walk this Way, I beg of you.

Tyr. — Sir —

Phi. Please to sit here, Sir, for you seem to be cold.

Tyr. Sir, I thank you ; I am a little cold, I confess.

Phi. Give me Leave, *Tyrunculus*, to repeat once more, that I am proud to see you ; and I hope, after the many Promises you have made, you are now come on Purpose to spend an Hour or two with me.

Tyr. Indeed, *Philomathes*, I came with that Design, if it be convenient ; if not, I beg I may wait upon you at a more suitable Opportunity.

Phi. Indeed, Sir, I shall consent to no such Thing ; you shall not put me off from Time to Time with your Apologies ; you see I am alone ; why then should you think it is inconvenient ?

Tyr. Pardon me, *Philomathes* ; as I see your Books before you, I thought you might be too busy at this Time.

Phi. Pish, — I am never so engaged with them, but that I'm always ready to receive my Friend ; though I confess (when I have not the Pleasure of an agreeable Companion) they are pretty Company to me of themselves.

Tyr. That I believe ; for come when I will, you are always upon the Study.

Phi. Indeed I am no close Student, *Tyrunculus* ; 'tis true, some things require more Thought and Consideration than others, and I believe none more than what I have now been reading.

Tyr. Pray what is that, if I may be so free ?

Phi. No Secrets at all, *Tyrunculus* ; you may look of any of the Books and welcome.

Tyr.

Tyr. So, so! *Algebra!* ——— *Cocker, Hammond, Saunderson, Kersey, Ward, De Billy, Wolfius, &c.* You have a Variety of Authors upon my Word.

Phi. The Science requires it, *Tyrunculus*.

Tyr. I believe so, and a fine Science it is; for my Part, I wish I understood it; but it is so hard for a Learner to step into it of himself, that I think it is quite tiresome.

Phi. You surprize me, *Tyrunculus*; why, you talk'd of learning long ago.

Tyr. I did so, and bought two or three good Pieces (as they are call'd) upon it; but I own I am very little the wiser for all my studying.

Phi. Do not say you studied; I fancy you only gave them a careless Look or two; and Things of this Nature should be read slow, and with the greatest Attention.

Tyr. I am sorry, *Philomathes*, you think I have taken no Pains; many an Hour's Rest have I broke, to learn only the first four Rules; and as for Equations, I never could understand; and to say the Truth, I have laid the Thoughts of it aside for some Months.

Phi. I beg, *Tyrunculus*, you would not be angry; what made me think so is, because I know most Learners are apt to run over Things too hastily, and then blame the Author for their not coming at them directly: However, it is plain the Fault is either in you or the Books (if not in both) that you have made no better Improvement.

Tyr. I am apt to think the Fault is in them as much as in me.

Phi. Perhaps so; but what signifies barely asserting it, without giving a Reason?

Tyr. Whether I be expert enough to give you a sufficient one, I will not say.

Phi. Let me hear you give some Reason or other, I beg.

Tyr. Why then, *Philomathes*, two of my Authors treat only of *Algebra* itself, beginning with *Problems* directly; the other indeed begins at *Addition*, and proceeds on to the first four *Algebraic* Rules, which I learnt pretty well; but when I came to *Algebraic Fractions*, I (not knowing any Thing of *Vulgar*, and he being so concise) could understand very little of them; on which Account, when I came to Equations, I was quite at a Loss; for I perceive there are very few but what have Fractions, and I know very little of them, except just to read them.

Phi. Why indeed, *Tyrunculus*, if you are not acquainted with *Vulgar Fractions*, it is in vain you pretend to study *Algebra*, for they are the very Basis and Foundation of it; however, *Algebraic Fractions* are done after the same Manner as *Vulgar*.

Tyr. But you will allow them to be much the harder of the two, I imagine?

Phi. Yes, yes, I grant it; for when the Learner is well acquainted with *Vulgar*, he will soon understand *Algebraic Fractions*; besides, it will save him a great deal of Trouble, for it is impossible to reduce an Equation, in order to discover the Value of the unknown Quantity, without understanding one or both of these.

Tyr. Since you grant this then, *Philomathes*, I think you must own the Fault to be in the Books rather than in me?

Phi. 'Tis true, the Books you speak of are not fit for Learners; but still, I must not indulge you so much as to lay all the Fault on them; for, as I said before, though they may not be immediately fit for Beginners, yet they may be very good Books; for you are to consider, *Tyrunculus*, that some Au-

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thors suppose Persons previously to understand such and such Rules.

Tyr. You say very right, *Philomathes*; but pray did you never hear any besides me complain of the Difficulty of learning *Algebra* by most Authors?

Phi. Yes, a great many, and there may be Reasons for it; but we should always, in such Cases, pass our Sentiments with Reason, Caution, and Tenderness, and not blame Authors on Account of every supine Learner; for it is evident we can have no better, if we go to the Extent of the Science; tho' I must confess thus far, that, upon the Rudiments, the Generality are a little dark, and too concise.

Tyr. Since you own this, *Philomathes*, how then could you so severely blame me, that I have made no better Progress?

Phi. You are not so much to be blamed as I thought for; but still, you are to be blamed in this Respect, that you have not consulted more Books; for he that designs to be the Master of any Art or Science, should certainly provide himself with a Sufficiency for such an Undertaking; for it often happens, that what one Author is deficient in, or treats darkly upon, another sets forth in a clear and easy Manner to be understood.

Tyr. A Variety of Books cannot be had, you know, without Expence.

Phi. You make me smile to hear you talk of Expence; you know very well you can afford to purchase any Books you have a Mind to.

Tyr. That may be, but I speak in Pity to those that cannot; for by Reason of this, many Minds lie quite uncultivated, which otherwise would make fine Improvements in several Branches of Learning: This was the Case of *Tyro*, when he took in Hand

to learn this Science, and I made him a Present of *Cocker* and *Hammond* to encourage him.

Phi. You did very kindly, and could not have made him a prettier Present; and he will certainly learn, for he has a very pretty Turn of Thought for Figures.

Tyr. I have heard *Novitius* say the same, (for they both practise together) but yet he says, that he never could rightly understand in what Manner *Cocker* reduces several of his Equations; for my Part, I wonder at his Patience.

Phi. Delight, *Tyrunculus*, Delight carries us thro' many Difficulties: But pray, do you remember any particular Question or Equation that he seems so much puzzled about?

Tyr. That I do not; but I heard him wish he had the Happiness of being acquainted with you, he would have ask'd you how to reduce a certain Equation or two, which puzzles him pretty much, but that he feared you would take it amiss.

Phi. Not I, in the least; you know, *Tyrunculus*, I am of no such selfish Temper; I hate it of all Things; it would be Ingratitude not to communicate that freely, which I received so: Base and sordid Spirits will indeed deny their Assistance, that they may have the Pleasure of laughing at the Ignorance of those that they ought to have instructed.

Tyr. It is very true, *Philomathes*, *Philautus* is of this unhapy Disposition; I heard *Novitius* say, a little while ago, he ask'd him only a single Question, and he would not resolve it, but gave him very little to answer, and seem'd, he said, to be affronted; and yet you know they are intimately acquainted, and he always expresses the greatest Regard for *Novitius* in other Respects.

Phi. It is very surprizing! The various Tempers of Men are not easily to be accounted for you know,
Tyr.

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Tyrunculus : To be sure, that Man can be of no Service to any Society, that is not ready and willing to assist every Member of it, and especially when he is entreated. *Lucretius* was wont to say, "That he would seek all Opportunities to communicate whatever he thought might be serviceable to any Man; and that, if Wisdom and Knowledge were given to him with that *Reserve*, that he might not impart it to others, he said he would much rather choose to be without them."

Tyr. ————— (smiles.)

Phi. What do you smile at, *Tyrunculus*?

Tyr. Nothing, Sir ————— I was only going to say I wish *Lucretius* liv'd near me.

Phi. That is not amiss, *Tyrunculus*, I confess: However, I am as ready to serve you as he would be; and to shew you I am, if you approve of it, and have a Mind to have a little Touch at *Algebra*, I will give you the best Instructions I am capable of, upon Promise you will apply diligently; for were I sure you would not, I should repent my Folly even in asking you, much more in the undertaking itself. You remember the old Proverb, *Strike the Iron while it is hot*: If you slight this Offer, you may, perhaps, afterwards blame yourself. ————— Come, what say you?

Tyr. Dear *Philomathes*, I am still more obliged to you; and as I am bound in Duty to accept of your Kindness with Thanks, give me Leave to say, I will use my best Endeavours not to frustrate your kind Benevolence; but indeed it is giving you too much Trouble.

Phi. Do not mention it: You are welcome, as I said before; but pray when will you begin, for the sooner the better, if you take my Advice?

Tyr. That must be as you please, Sir.

Phi.

Phi. It never can be more convenient than now, as we are alone, and free from any Interruption.

Tyr. With all my Heart, Sir.

Phi. Well then, *Tyrunculus*, I would have you observe the Method I shall take for your Instruction: I shall first begin with you at *Vulgar Fractions*, (as you have, you say, but very little Notion of them) and shall treat more of them than is required in the *Algebraic* Part, that you may see their Use in other Respects: Then I shall proceed to *Algebraic Fractions*, the Rule of *Proportion*, and *Equations*; wherein I shall give you several Examples very rarely to be met with, or so easily demonstrated: After these, I shall make some necessary Observations, and proceed directly to *Algebraic Problems*; and shall work them so gradually, that you cannot miss (if you take any Pains at all) to understand every Operation. But as several Things will, no Doubt, happen, that you may not immediately, upon first reading, have a true Notion of; pray do so much Justice to yourself, as to ask the Meaning of every Thing you are at a Loss for, and do not content yourself to go away half taught. For my Part, I shall be careful to avoid any Thing that I think may give Occasion to stop you in the Undertaking: Be you but as diligent to observe the Rules and Examples, and you will soon be Master of *that* which I shall hereafter instruct you in; and then *Tyrunculus*, *Cocker* and *Hammond* will at once appear both beautiful and easy to you; or *Saunderson* and *Kersey*, if you think fit to purchase them, (the former of which I more particularly recommend:) But if you have not the Happiness to meet with either of these, you are here qualified for *Ward*, and have a good Foundation to peruse more concise Authors, such as *De Billy*, *Gestinus*, *Wolffius*, &c. &c.

DIALOGUE II.

SECT. I.

On NOTATION and REDUCTION of VULGAR FRACTIONS.

Tyr. WHAT do you mean by Notation of Fractions?

Phi. Notation shews you how to write down and express any Fraction.

Tyr. What is a *Vulgar Fraction*?

Phi. A *Fraction* signifies a broken Number, or, in other Words, when Unity, or the Number 1, is divided into any Number of Parts, those Parts are a *Fractional Part* of the Integer itself, and is called a *Vulgar Fraction*.

Tyr. How am I to know a *Vulgar Fraction*?

Phi. Whenever you see any Figure or Figures, with other Figure or Figures underneath, and a Dash between them, (thus, $\frac{3}{4}$ $\frac{5}{6}$) they are *Vulgar Fractions* of some Denomination or other.

Tyr. What! Are there different Sorts then?

Phi. Yes, three at least.

Tyr. Tell me their different Names, if you please?

Phi. First then, there are *Simple*, *Single*, or *Proper Fractions*, (for you must note they are frequently called by either of these Names;) 2dly, *Improper*; and, 3dly, *Compound Fractions*.

Tyr.

Tyr. How are these separately known, or express'd in Figures?

Phi. Thus; $\frac{2}{3}$, $\frac{5}{9}$, $\frac{14}{17}$, $\frac{412}{716}$, &c. are all *Simple Fractions*; they are so called because each of the Numerators are less than the Denominators belonging to them.

Tyr. What do you mean by Numerator and Denominator?

Phi. The Numerator always stand a-top of the Dash, and the Denominator underneath: Thus, in the foregoing *Simple Fractions*, 2, 5, 9, and 412, are Numerators; and 3, 8, 14, and 716, are their respective Denominators.

Tyr. Very well: What is an *Improper Fraction*?

Phi. *Improper Fractions*, contrary to *Simple* ones, have their Numerators larger than the Denominators: Thus, $\frac{4}{3}$, $\frac{13}{7}$, $\frac{63}{9}$, and $\frac{716}{49}$, &c. are *Improper Fractions*.

Tyr. What do you mean by *Compound Fractions*?

Phi. *Compound Fractions* are *Fractions* of *Fractions* compounded, coupled, or joined together, by the Word *of*, Thus, $\frac{2}{3}$ of $\frac{4}{5}$, or $\frac{4}{5}$ of $\frac{2}{3}$ of $\frac{21}{8}$, &c. are all *Compound Fractions*: Do you understand it?

Tyr. Yes, surely: But how are these different *Fractions* read, or verbally express'd?

Phi. Thus, $\frac{2}{3}$ and $\frac{14}{17}$, is two Thirds and fourteen Twenty-ninths; also $\frac{14}{5}$ is 14 Fifths, and $\frac{1}{5}$ of $\frac{1}{3}$ of $\frac{17}{8}$, 3 Fourths of 5 Sixths of 7 Elevenths, &c.

Tyr. The 3 Fourths of the 5 Sixths of the 7 Elevenths: But of what?

Phi. Why, the $\frac{1}{5}$ of the $\frac{1}{3}$ of the $\frac{17}{8}$ Parts of an Integer, or whole Number.

Tyr. I ask Pardon: But this must be very hard to tell that.

Phi.

Phi. You are not to concern yourself about this at present; you will find it easy enough by-and-by.

Tyr. Are there no more Fractions?

Phi. Properly speaking there are not; but there is what we call a *Mixt Number*.

Tyr. What is that pray?

Phi. A *Mixt Number* consists of two Parts, the first Part a whole Number, and the other a *Fraction*: Thus, $4\frac{3}{5}$, and $247\frac{1}{2}$, are *Mixt Numbers*; that is, 4 whole Numbers and $\frac{3}{5}$ of Unity or 1, &c.

Tyr. I understand it very well: What is the next Thing you purpose?

Phi. Nothing more concerning the Names of *Fractions*: I shall now give you three or four Observations, which you will do well to remember.

OBSERV. 1.

The Value of every *Simple Fraction* is less than Unity or an Integer, by so many Times as the Numerator is contained in the Denominator; as you will see demonstrated (*Case 9*) in *Reduction*: So also is the Value of all *Compound Fractions*, if they be compounded of *Simple* ones; for they are all but one *Simple Fraction* when reduc'd, as you will see (*Case 6*) in *Reduction*.

OBSERV. 2.

Contrary to these, the Value of any *Improper Fraction* is more than an Integer, or as many whole Integers as the Denominator is contained Times in the Numerator; (See *Case 9* in *Reduction*.)

OBSERV. 3

When the Numerator and Denominator are alike, this is called by some an *Improper Fractions*, but with what

what Propriety I know not, seeing it is only Unity itself: For $\frac{1}{4}$ of a £. Sterling is 1 £. and $\frac{3}{4}$ of a Yard 1 Yard; because 4, divided by 4, &c. make one whole Integer.

OBSERV. 4.

When you would make any whole Number into an *Improper Fraction*, then put Unity underneath it. Thus 4 will be $\frac{4}{1}$, and 126 will be $\frac{126}{1}$, &c. Pray remember this.

Tyr. I understand you quite well; but pray how is the Value of a *Fraction* discovered, in Order to know what Relation it bears to an Integer?

Phi. *Fractions* are reduced by certain Rules or Cases in *Reduction*, of which if you be Master, you will soon *add, subtract, multiply, and divide*; but not else.

Tyr. Why is *Reduction* taught before the others pray?

Phi. Because the *Fractions* must be first reduced before you can do the other Rules: *Reduction* therefore prepares the *Fractions* as you will see by the following Examples.

S E C T. II.

REDUCTION of VULGAR FRACTIONS.

Tyr. I AM mightily pleased with what you have shewn me concerning the Nature of *Vulgar Fractions* ; but I think long to know how to reduce them.

Phi. That you shall directly ; but are you sure you know what a *Simple, Compound, Improper Fraction*, and *Mixt Numbers* are ? for they must be known : And if you think you do not understand them quite well, give them another Look ; you cannot be too perfect.

Tyr. I am positive I understand what they mean.

Phi. Very well : Pray hand me that Slate then ; I will reduce them in their Order before your Face, and you may try at other Examples, which you may set yourself at your Leisure.

C A S E I.

To reduce a Mixt Number to an Improper Fraction.

The Rule is,

Multiply the whole Number by the Denominator of the *Fraction* belonging to it, and take in the Numerator ; then under this Product set the Denominator ; so is this *Improper Fraction* equivalent to the *Mixt Number* given.

REDUCTION of

EXAMPLE 1.

Reduce $4\frac{2}{3}$ to an Improper Fraction.

$$\begin{array}{r}
 4\frac{2}{3} \\
 5 \\
 \hline
 22 \\
 \hline
 5 \text{ Ans.}
 \end{array}$$

EXAMPLE 2.

Reduce $51\frac{2}{11}$ to an Improper Fraction.

$$\begin{array}{r}
 51\frac{2}{11} \\
 11 \\
 \hline
 563 \\
 \hline
 11 \text{ Ans.}
 \end{array}$$

EXAMPLE 3.

Reduce $576\frac{11}{24}$ to an Improper Fraction

$$\begin{array}{r}
 576\frac{11}{24} \\
 24 \\
 \hline
 2308 \\
 1153 \\
 \hline
 13836 \\
 \hline
 24 \text{ Ans.}
 \end{array}$$

Tyr.

VULGAR FRACTIONS. 15

Tyr. He that can do common Multiplication may do this.

Phi. True; and he that can do common Division may do the next, it being only the Reverse of the former Case, as you'll see by the same Examples.

CASE 2.

To reduce an Improper Fraction to its equivalent, whole, or mixt Number.

Rule is,

Divide the Numerator by the Denominator, and if any Thing remains, set it over the Denominator, for a new Numerator, and it is done.

EXAMPLE 1.

Reduce $2\frac{2}{3}$ to its equivalent, whole, or mixt Number.

$$\begin{array}{r} 5 \overline{) 22} \\ \underline{} \end{array}$$

$4\frac{2}{3}$ *Ans. See Ex. 1, Case 1.*

EXAMPLE 2.

Reduce $10\frac{1}{11}$ to its equivalent, whole, or mixt Number.

$$\begin{array}{r} 11 \overline{) 563} \\ \underline{} \end{array}$$

$51\frac{2}{11}$ *Ans.*

C 2

Ex-

EXAMPLE 3.

Reduce $138\frac{38}{120}$ to its equivalent, whole, or mixt Number.

$$24)13838 \text{ (} 576\frac{14}{120} \text{ Anf.}$$

120

183

168

158

144

14

Note, When there is no Remainder in the Division, then will the whole Number be equivalent, or equal to the given *Improper Fraction*. As for Example: Suppose I would reduce $56\frac{56}{7}$ to its Equivalent, I divide 56 by 7, and the Quotient is 8; so is 8 equal to $56\frac{56}{7}$; so also $2\frac{28}{7}$ is equal to 19. This is easily seen by the next Case.

CASE 3.

To reduce any whole Number to an *Improper Fraction*.

Rule is,

Multiply the whole Number by any Figure at Pleasure, and under the Product set the same Figure you multiply'd by, and you have an *Improper Fraction* equal to the given whole Number.

Ex-

VULGAR FRACTIONS. 17

EXAMPLE 1.

Reduce 14 to an Improper Fraction.

$\begin{array}{r} 14 \\ 5 \\ \hline 70 \\ \hline \text{Ans.} \\ 5 \end{array}$	$\begin{array}{r} 14 \\ 9 \\ \hline 126 \\ \hline \text{Ans.} \\ 9 \end{array}$	$\begin{array}{r} 14 \\ 12 \\ \hline 168 \\ \hline \text{Ans.} \\ 12 \end{array}$
--	---	---

Here you see I multiply the whole Number by 5, by 9, or by 12, or any other Figure, and the *Improper Fractions* are all equal to each other, and are also equal to 14. Therefore this is an unlimited Question, to which an infinite Number of Answers may be given, and all right; but if the Question be proposed thus, it will be limited, and can then have only one Answer. As for Example.

EXAMPLE 2.

Reduce 14 to an Improper Fraction, whose Denominator shall be 15.

Here as I am to have 15 for its Denominator, I am oblig'd to multiply by 15, and no other Figure.

$$\begin{array}{r} 14 \\ 15 \\ \hline 70 \\ 14 \\ \hline 210 \\ \hline \text{Ans. equal to 14, as before.} \end{array}$$

C 3

CASE

CASE 4.

To reduce a Fraction to its lowest Terms, equal in Value to the Fraction given.

Rule is,

Divide the Numerator and Denominator by any Figure that will divide them both without any Remainder, and continue so doing till you can divide them no lower; so will this last Quotient be the lowest Terms equal to the original given Fraction.

EXAMPLE I.

Reduce $\frac{144}{216}$ to its lowest Terms.

Divisors	2	3	4	3	
Num.	144	72	24	6	2
Denom.	216	108	36	9	3

Ans. $\frac{2}{3} = \frac{144}{216}$

EXAMPLE 2.

Reduce $\frac{576}{960}$ to its lowest Terms.

Divisors	6	8	4	
Num.	576	96	12	3
Denom.	960	160	20	5

Ans. $= \frac{3}{5}$

Do you understand the Work?

Tyr. I understand all very well, but the two Lines you make after the Answers I don't rightly apprehend.

Phi.

VULGAR FRACTIONS. 19

Phi. What this Mark (=) do you mean?

Tyr. Yes.

Phi. It is the Sign of Equality, it signifies that $\frac{2}{3}$ is equal to $\frac{14}{21}$; and $\frac{3}{4}$ is equal to $\frac{27}{36}$. You will frequently see it used by and by.

Note. After the same Manner are *Algebraic Fractions* abbreviated. For suppose I were to abbreviate

or reduce — to its lowest Terms; it is only taking away such Letters or Quantities as are alike out of the Numerator and the Denominator, and the Work is done. Thus in the above *Algebraic Fraction*—

I find *b* both in the Numerator and Denominator, therefore by taking *b* from both, I have in — its lowest Terms = —. But this you will see more of in Case 4 of *Algebraic Fractions*.

Tyr. Is there no other Method of reducing a *Fraction* to its lowest Terms, because it is difficult to find Figures that will divide some *Fractions*.

Phi. 'Tis true, for that Figure which will divide one, will not perhaps divide the other, therefore there is a Way to tell what Figure will do it at one Operation.

Tyr. That must be mighty pretty, pray let's see it?

Phi. You shall.

CASE

CASE 5.

Another Way to reduce a Fraction to its lowest Terms at one Work. (Euc. 7, Pr. 1, 2, 3.)

Divide the Denominator of the *Fraction* by its Numerator, and if any Thing remains, divide your former Divisor by it, and if any Thing yet remains, divide your last Divisor by *that*; thus proceed till you have Nothing remain, and then shall your last Divisor be a *Common-Measurer*, that will infallibly divide both the Numerator and Denominator of the given *Fraction* into its lowest Terms at one Work.

Tyr. Pray give me an Example, and explain it in Words.

Phi. I will.

EXAMPLE I.

Reduce $\frac{147}{252}$ to its lowest Terms by a Common-Measurer.

First, I divide 252 by 147 the Numerator, and it goes once, and 105 remains over; this 105 I make now a Divisor, and the last Divisor (*viz.* 147) a Dividend, and find it contains once 105, and 42 remains over; by this 42 I divide the last Divisor 105, and find 21 remains; and lastly, by this 21 I divide the last Divisor 42, and find Nothing remains over: So is the last Divisor 21 a *Common-Measurer*, that will reduce the given *Fraction* $\frac{147}{252}$ to its lowest Terms at once; for dividing the Numerator 147 by 21, I have 7 for a new Numerator; and dividing 252 by 21, I have 12 for a new Denominator; and thus I find $\frac{7}{12} = \frac{147}{252}$.

Ex.

VULGAR FRACTIONS. 21

EXAMPLE 2.

Reduce $\frac{3}{4}$ to its lowest Terms by a Common-Measurer.

$$\begin{array}{r} 574) 861 \text{ (1)} \\ \underline{574} \end{array}$$

$$\begin{array}{r} \text{Com. Measure } 287) 574 \text{ (2)} \\ \underline{574} \\ 0 \end{array}$$

$$\begin{array}{r} \text{Num.} \\ 287) 574 \text{ (2 New Num.)} \\ \underline{574} \\ 0 \end{array}$$

$$\begin{array}{r} \text{Denom.} \\ 287) 861 \text{ (3 New Denom.)} \\ \underline{861} \\ 0 \end{array}$$

$$\text{Ans. } \frac{3}{4} = \frac{3}{4}.$$

Tyr. I understand it well ; but pray suppose a *Fraction* cannot be abbreviated by a *Common-Measurer*, by its proving an Unit at last ?

Phi. Why then it is in the lowest Terms already : Such a one is $\frac{1}{1}$.

Tyr. Very well. Pray are there no more Cases in Reduction ?

Phi. Yes, here follows,

CASE

Ex.

CASE 6.

To reduce a Compound Fraction to a Simple-one of the same Value.

Rule is,

Multiply all the Numerators one into another for a new Numerator ; then multiply all the Denominators together for a new Denominator, so shall this new Fraction be equal to the Compound Fraction given.

Tyr. This is so easy I think I can do it directly ; pray try me ?

Phi. No Doubt, for it is only common Multiplication.

EXAMPLE I.

Reduce $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{1}{4}$ to a Simple Fraction.

Tyr. I set down all the Numerators thus, 2

	5
	—
Then I multiply all the Denom.	3
	10
	6
	—
	3
	—
	18
	—
	4
	—
	72
	—

30 New N.

New Denom.

Ans. $\frac{10}{72} = \frac{5}{36}$ of $\frac{1}{4}$ of $\frac{1}{2}$.

Phi.

VULGAR FRACTIONS. 23

Phi. It is very right, *Tyrunculus*; but
What this there is a Character used for Multipli-
× Cross cation which will mightily shorten the
signifies. Work, and take up less Room; besides
 it is frequently used in *Algebra*. This is it ($\times$), and
 it signifies that all the Numbers between which it
 stands are to be multiplied together. Thus $4 \times 6 \times 2$
 $= 48$. Pray remember it.

Tyr. I know your Meaning immediately. Thus
 4 multiplied by 8 is 4×8 , that is 32. So 3×2
 $\times 5 = 30$. Is it so or not?

Phi. You are very right; now I'll try you with
 another Sum.

EXAMPLE 2.

Reduce $\frac{5}{7}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{11}{12}$ to a Simple Fraction.

Tyr. I multiply all the Numerators together,
 Thus $5 \times 4 \times 3 \times 11 = 660$ for a new Nu-
 merator.

And $7 \times 5 \times 4 \times 12 = 1680$ for a new De-
 nominator.

So is $\frac{660}{1680} = \frac{5}{7}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{11}{12}$ *Ans.*

Phi. It is very well done *Tyrunculus*, now we will
 proceed to

CASE 7.

*To reduce Fractions of unequal, or different Denomina-
 tors, to Fractions of the same Value, having but one
 common Denominator to all the Numerators.*

Rule is,

Multiply all the Denominators together for a com-
 mon Denominator; then take each Numerator, (be-
 ginning

ginning at the first) and multiply it into all the Denominators except its own Denominator; so shall these different Products be new Numerators to the common Denominator, equal to that *Fraction* whose Numerator you multiplied into the Denominators, which placed over the common Denominator, and the Work is done. Do you think you could do this directly?

Tyr. No, this is not so easy as the last Case. Pray give me one Example at large, and then I'll try?

What N. N. *Phi.* I will. Pray remember and C. D. N. N. signify new Numerator, and signify. C. D. common Denominator.

EXAMPLE I.

Reduce $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{8}$, to Fractions of the same Value, having a common Denominator.

$$\frac{2}{3}, \frac{3}{5}, \frac{5}{8}.$$

First $\frac{2}{3}$	then $\frac{3}{5}$	and $\frac{5}{8}$	and $\frac{5}{8}$
$\frac{2}{3}$	$\frac{3}{5}$	$\frac{5}{8}$	$\frac{5}{8}$
$\frac{15}{8}$	$\frac{10}{8}$	$\frac{9}{8}$	$\frac{25}{8}$
C.D. 120	N. N. 80	N. N. 72	N. N. 75

Ans. $\frac{80}{120} = \frac{2}{3}$, for 80 is $\frac{2}{3}$ of 120. Also $\frac{72}{120} = \frac{3}{5}$ and $\frac{75}{120} = \frac{5}{8}$.

Tyr. 'Tis so plainly done, that I think I can do another Example.

Phi. Possibly you may, as you take good Observation.

Ex-

VULGAR FRACTIONS. 25

EXAMPLE 2.

Reduce $\frac{1}{3}$, $\frac{2}{5}$, $\frac{7}{8}$, and $\frac{3}{4}$, to Fractions having a common Denominator.

Tyr. First then, $5 \times 6 \times 8 \times 4 = 960$ for a common Denominator. Then $4 \times 6 \times 8 \times 4 = 768$ N. N. And $5 \times 5 \times 8 \times 4 = 800$ N. N. Again, $7 \times 6 \times 5 \times 4 = 840$ N. N. And lastly, $3 \times 8 \times 6 \times 5 = 720$ N. N. These new Numerators plac'd over the common Denominator, I find the Answer to be $\frac{768}{960} = \frac{1}{3}$, $\frac{800}{960} = \frac{2}{5}$, $\frac{840}{960} = \frac{7}{8}$, and $\frac{720}{960} = \frac{3}{4}$. Is it right?

Pbi. You surprize me, to see you so apt; see what Care is! You have no Occasion for more Examples in this Case. We will pass on then to

CASE 8.

To reduce Fractions of one Denomination to another.

This consists of two Parts, *ascending* or *descending*. And first of *ascending*.

When a *Fraction* is given to be brought from a less to a greater Denomination, then set down the *Fraction*, and make a compound one of it, according to the Denomination it is to be brought into; and this *compound Fraction* is made by considering how many of the less make one of the greater; then reduce this *compound* to a *simple Fraction*, and you will have a *Fraction* of another Denomination, equal in Value to the given *Fraction*.

Tyr. This is a hard Case, this is not understood by bare reading.

Pbi. It is harder than some of the rest; but an Example or two will make it plain.

D

Ex-

EXAMPLE I.

Reduce $\frac{3}{4}$ of a Penny to the Fraction of a £. Sterling.

Now observe, as 12 Pence make a Shilling, and 20 Shillings a Pound, I make a *compound Fraction* of $\frac{3}{4}$ thus,

$$\frac{3}{4} \text{ of } \frac{1}{12} \text{ of } \frac{1}{20} \text{ of a } \text{£}.$$

Now this reduced to a *simple Fraction*, viz. $3 \times 1 \times 1 = 3$ N. N. and $5 \times 12 \times 20 = 1200$ N. D. So is $\frac{3}{1200}$ of a £. = $\frac{1}{400}$ of a Penny.

O R,

Otherwise make a *compound Fraction* of it at once; that is, 240 Pence make a £. Sterling. Then it will be $\frac{3}{4}$ of $\frac{1}{240}$; this reduced to a *simple Fraction*, is $\frac{3}{1200}$ of a £. = $\frac{1}{400}$ of a Penny as above.

EXAMPLE 2.

Reduce $\frac{3}{4}$ of a Farthing to the Fraction of a Guinea.

This will be $\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{21}$ of a Guinea.

Now $3 \times 1 \times 1 \times 1 = 3$ N. N. and $4 \times 4 \times 12 \times 21 = 4032$ N. D. So is $\frac{3}{4032}$ of a Guinea = $\frac{1}{1344}$ of a Farthing.

2. DESCENDING.

In *descending* you are to consider, that the *Fraction* is to be brought from a greater to a less Denomination; therefore, as you multiplied the Denominator of the *given Fraction* by the Parts contained in the Integer in Reduction *ascending*, so now here you must multiply the Numerator of the *given Fraction* by the same

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same Parts, and you have the Answer. Or, which is all the same, only invert the Parts contained in the Integer, (that is, turn them topsy-turvy) and make of them a *compound Fraction* as before, then reduce it to a *simple Fraction*, and it is done.

EXAMPLE I.

Reduce $\frac{4}{3}$ of a £. Sterling to the Fraction of a Penny.

Here I consider that a Shilling is $\frac{1}{20}$ of a £. and a Penny $\frac{1}{12}$ of a Shilling, therefore I multiply the Numerator 4 by 20 and by 12, and the Product is 960, which I place over the Denominator, thus $\frac{240}{960}$. So is $\frac{240}{960}$ of a Penny equal to $\frac{4}{3}$ of a £. or 16 Shillings.

Or, by inverting the Parts as above directed, it will be $\frac{4}{3}$ of $\frac{1}{12}$ of $\frac{1}{20}$ a *compound Fraction*, which reduced to a *simple one*, viz. $4 \times 20 \times 12 = 960$ N. N. and $5 \times 1 \times 1 = 5$ N. D. To prove this we will try *Example 2.* of Reduction ascending.

EXAMPLE 2.

Reduce $\frac{4032}{3}$ of a Guinea to the Fraction of a Farthing.

Here $\frac{4032}{3}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$.

Now $3 \times 21 \times 12 \times 4 = 3024$ N. N.

And $4032 \times 1 \times 1 \times 1 = 4032$ N. D. This *Fraction* abbreviated is $= \frac{1}{4}$ of a Farthing. So is $\frac{1}{4}$ of a Farthing $= \frac{1}{4032}$ of a Guinea, as in *Example 2* of last Rule.

What do you say to this Case, *Tyrunculus*?

Tyr. I think I understand it pretty well; however, I will look it over again, and try at other Examples.

Phi. Do so. Now, *Tyrunculus*, we are come to the

D. 2

most useful and pleasant Case of all, which is to find the true Value of any *Fraction*.

Tyr. That I shall like I know.

Phi. Pray observe carefully the Rules and Examples, and I dare say you will work any of them directly after me.

CASE 9.

To find the Value of a Fraction in Money, Weight, or Measure,

Rule is,

Multiply the Numerator by the Parts contain'd in the Integer to which it belongs, always observing to begin with that Part nearest related to the Integer; then divide by the Denominator, and if any Thing remains, multiply it by the next greatest Part nearest related to the Integer, and divide again by the Denominator. Thus proceed till you can reduce it no lower for Want of Parts in the Integer, and the Work is done.

Tyr. I must beg one Example at large.

Phi. You shall, and you will need no more to understand the Case.

EXAMPLE I.

What is the Value of $3\frac{1}{2}$ of a £. Sterling?

First, in Order to find the Value of this *Fraction*, I consider the next nearest Part related to a £. and I find it to be Shillings. Now because 20 Shillings make a £. I multiply the Numerator 3 by 20, and it is 60, which I divide by the Denominator 32, and have 1 in the Quotient, which is 1 Shilling, and 28 remains over; this I call $\frac{28}{32}$ of a Shilling. Now as 12 Pence make a Shilling, multiply 28 the Numerator

VULGAR FRACTIONS. 29

erator by 12, and it makes 336, which I divide also by the Denominator 32, and the Quotient is 10, which is 10 Pence, and 16 remains over; this I call $\frac{1}{2}$ of a Penny; then as 4 Farthings make a Penny, I multiply the Numerator 16 by 4, and it is 64, which I divide again by the Denominator 32, and the Quotient is 2 Farthings, So that I find $\frac{1}{4}$ of a £ . to be 1s. 10d. $\frac{1}{2}$. See the Work.

$$\begin{array}{r} 3 \\ 20 \\ \hline 32) 60 \text{ (1s} \\ 32 \\ \hline 28 \\ 12 \end{array}$$

$$\begin{array}{r} 32) 336 \text{ (10d.} \\ 32 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 4 \\ \hline 32) 64 \text{ (2 grs.} \\ 64 \\ \hline 0 \end{array}$$

Ans. 1s. 10d. $\frac{1}{2}$.

D 3

What

What is the $\frac{4}{23}$ of a Moidore?

$$\begin{array}{r}
 4 \\
 27 \\
 \hline
 25) 108 \text{ (4 s.} \\
 100 \\
 \hline
 8 \\
 12 \\
 \hline
 \text{Ans. 4 s. 3 d. } \frac{3}{4} \text{ } \frac{3}{23} \\
 25) 96 \text{ (3 d.} \\
 75 \\
 \hline
 21 \\
 4 \\
 \hline
 25) 84 \text{ (3 qrs.} \\
 75 \\
 \hline
 9
 \end{array}$$

Questions to be tried.

3. What's the Value of $\frac{4}{23}$ of an Cwt. } *Ans.* 1 qr. 9 lb 5 oz. 5 drs. $\frac{1}{2}$
 4. What's the Value of $\frac{1}{317}$ of a Year? } *Ans.* 146 d. 5 h. 31 m. 36 s. $\frac{16}{317}$

☞ To find the Value another Way.

Suppose I wanted to know the $\frac{4}{23}$ of a £. or the $\frac{4}{23}$ of a Moidore.

This

VULGAR FRACTIONS. 31

This is done only by abbreviating the Numerator, and bringing it down to Unity; then will the *Fraction* be $\frac{1}{12}$ of a £. then multiply the Value by the Numerator, and you have its true Value.

First, What is the $\frac{1}{12}$ of a £. Sterling?

I abbreviate the Numerator to Unity or 1, and considering what $\frac{1}{12}$ of a £. is, I find it to be 1 s. 8 d. this I multiply by 5 the Numerator, and it makes 8 s. 4 d. which is the Value of $\frac{5}{12}$ of a £. Sterling. Again,

What is the $\frac{1}{8}$ of a Moidore?

I consider that $\frac{1}{8}$ of a Moidore is 1 s. 6 d. then by multiplying by the Numerator 7, I have 10 s. 6 d. which is the Value of $\frac{7}{8}$ of a Moidore.

N. B. I told you in *Dial.* the 2d, *Observ.* 1, that every *simple Fraction's* Value was less than an Integer or Unity, and that the Value of an *improper Fraction* was more. (*Observ.* 2.) To prove which let us take any two *Fractions*, one *simple*, and the other *improper*, and see what their Value is in Relation to a £. Sterling.

EXAMPLE I.

Suppose the Simple Fraction to be $\frac{1}{2}$ of a £.

I find $\frac{1}{2}$ of a £. to be 1 s. 8 d. therefore $\frac{1}{2}$ is 9 Times 1 s. 8 d. = 15 s. Now 15 s. wants 5 s. of the whole Integer or 1 £. therefore the *simple Fraction* $\frac{1}{2}$ is less in Value than the whole Integer or Unity, by 5 Shillings. Now contrary to this, an

improper Fraction's Value is more than the Integer itself; and its Value is greater or less according to the Largeness of its Numerator.

EXAMPLE 2.

What is the $2\frac{1}{2}$ of a £. Sterling?

This being an *improper Fraction*, I divide the Numerator 25 by the Denominator 4, and it gives 6 whole Integers, or 6 £. now as one remains over, it is $\frac{1}{4}$ of a £. which is 5 s. So I find $2\frac{1}{2}$ of a £. to be 6 £. 5 s. 10 d. Do you understand it, *Tyrunculus*?

Tyr. Had you only said it I might have been at a Loss; but you have demonstrated it so plainly, that I must be quite dull of Apprehension not to see the Nature of it.

Phi. I am glad you understand me; and pray do you think you understand all the 9 Cases in *Reduction* so as to work them now off Hand; for if you do not (at least all but the 8th, that being not so much wanted) I freely tell you that you will be at a great Loss; for the next four Rules depend wholly upon a true Knowledge of *Reduction*.

Tyr. You do well, *Philomathes*, to take such Care of me, and I hope every Learner will take your Advice; but for my own Part, I can safely say I understand *Reduction* quite perfectly.

Phi. Well, if so, we will proceed directly to *Addition*.

DIALOGUE III.

Of ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION, of VULGAR FRACTIONS.

SECT. I.

Of ADDITION of VULGAR FRACTIONS.

Tyr. HOW is *Addition of Fractions* performed?
 Phi. By this one general Rule, viz. all *compound Fractions* must first be reduced to *simple ones*, and all *Fractions* to a common Denominator, (by *Case the 6th and 7th of Reduction*;) then add all the Numerators together as in common *Addition*, and place their Sum over the common Denominator; and if it be an *improper Fraction*, reduce it to a mixt Number (by *Case 2. in Reduction*) and you have the Sum of all the *Fractions*.

EXAMPLE I.

Add $\frac{2}{5}$, $\frac{1}{5}$, and $\frac{4}{5}$ together.

Here because the *Fractions* have all one common Denominator, I only add the Numerators 2, 1, and 4 together, and their Sum is 7, which I place over the common Denominator 5, and the Sum is $\frac{7}{5}$ an *improper Fraction*, equal to $1\frac{2}{5}$ Ans.

EXAMPLE 2.

Add $\frac{5}{21}$, $\frac{6}{21}$, $\frac{14}{21}$ and $\frac{19}{21}$ together.

Ans. $\frac{44}{21} = 2\frac{2}{21}$.

Tyr.

Tyr. This is mighty easy; the Rule is so plain one cannot well miss.

Phi. Now I will set you a Question, *Tyrunculus*.

EXAMPLE 3.

Add $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$ together.

Tyr. I am afraid you have puzzled me; but stay a little, let me find it out myself. — I see how it must be done, I must reduce the *Fractions* first to a common Denominator, and then add the Numerators together as you have done above. Must I not?

Phi. You have it I perceive.

Tyr. First then, to reduce the *Fractions* to a common Denominator.

$\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$

I multiply $3 \times 2 \times 4 \times 6 = 144$ C. D. Then $2 \times 2 \times 4 \times 6 = 96$ N. N. Then $1 \times 3 \times 4 \times 6 = 72$ N. N. Again, $3 \times 2 \times 3 \times 6 = 108$ N. N. Lastly, $5 \times 4 \times 2 \times 3 = 120$. So that I find the new Numerators are as follows:

N. Numerators.

	96
C. D. 144	72
	108 and
	120
	—

Their Sum 396 which placed over the

common Denominator 144, stands thus, $\frac{396}{144}$; this, by *Case* the 2d. in *Reduction*, $= 2 \frac{108}{144}$, that is, $2 \frac{3}{4}$, the Sum of $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$.

Phi. It is quite right, *Tyrunculus*, you see what it is to mind the Rules given for Instruction; you will

VULGAR FRACTIONS. 35

will do *that* already which will cost a careless Reader ten Times the Trouble. Come, I'll try you with another.

Tyr. With all my Heart.

EXAMPLE 4.

Phi. Add $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ and $\frac{5}{8}$ of $\frac{3}{4}$ together.

Try. Let me see: I must first (by *Case 6. of Reduction*) reduce the *compound* to *simple Fractions*, and then the *Fractions* to a common Denominator, and proceed as before. First then I multiply all the Numerators together, *viz.* $2 \times 4 \times 3 = 24$ N. N. Then $4 \times 5 \times 4 = 80$ C. D. So is $\frac{24}{80} = \frac{3}{10}$ of $\frac{4}{5}$ of $\frac{3}{4}$. Then the other *compound Fraction*, *viz.* $\frac{5}{8}$ of $\frac{3}{4}$, reduced, is $= \frac{15}{32}$. So the two *simple Fractions* to be added are $\frac{24}{80}$ and $\frac{15}{32}$. These I reduce to a common Denominator, as in *Example 3*, and find them to be $\frac{1152}{3840}$ and $\frac{1600}{3840}$. Then have I nothing to do but add the Numerators together, and their Sum is 2752, which I place over the common Denominator thus, $\frac{2752}{3840}$, and this I apprehend to be the Sum of $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ and $\frac{5}{8}$ of $\frac{3}{4}$. Is it so or not?

Phi. It is, and I am proud to see you so well grounded in the Rules of Reduction: However, I think I can pose you the very next Question. Will you try at it?

Tyr. To be sure I will; for I imagine you will shew me if I cannot do it.

Phi. You need not doubt it—Come then.

EXAMPLE 5.

Add 4 £. $\frac{3}{4}$, 7 £. $\frac{3}{4}$, of $\frac{3}{4}$ and $1\frac{1}{2}$ together.

Tyr. _____

Phi. I thought I should puzzle you; however, be not discouraged.

Tyr.

Tyr. There is Nothing that puzzles me but the last *Fraction* $\frac{5}{12}$, for I am not certain whether it belongs to the *compound Fraction* that stands before it, or whether it be a separate *Fraction* by itself.

Phi. It cannot belong to the *compound Fraction*, because the Word *of* is not between them. There are two Ways to do this and such like Questions; but the second is the shortest and easiest Method in my Opinion. I shall therefore only tell you the Way to work it, and leave you to try it by yourself at large*. Observe then,

METHOD I.

Reduce the *compound Fraction* $\frac{3}{4}$ of $\frac{2}{3}$ to a *simple-one*, which is $\frac{6}{12}$. Then will the Sum be thus, Add 4 £. $\frac{3}{4}$, 7 £. $\frac{6}{12}$ and $\frac{5}{12}$ together. Now by reducing the *mixt Numbers* 4 $\frac{3}{4}$ and 7 $\frac{6}{12}$ into *improper Fractions*, I have $\frac{19}{4}$ and $\frac{11\frac{1}{2}}{1}$. Then may the Sum be read thus; add $\frac{19}{4}$, $\frac{11\frac{1}{2}}{1}$ and $\frac{5}{12}$ together. These *Fractions* I reduce to a common Denominator (as in *Example 3.*) and find them to be $\frac{34\frac{20}{20}}{720}$, $\frac{5\frac{1}{2}8}{720}$ and $\frac{300}{720}$. These Numerators added together, and divided by the common Denominator 720, gives 12 in the Quotient, and 408 remaining over. So is the Answer 12 £. $\frac{408}{720} = 12$ £. 11s. 4d.

METHOD 2.

The second Way certainly is the best, because you have no Business to meddle with the whole Numbers, but only with the *Fractions*, and then add their Sum to the whole Numbers afterwards.

The

* It is supposed, that the Learner by this Time knows how to reduce mixt Numbers to *improper Fractions*, and *Compound* to *Simple-ones*, having had so many Examples of both Kinds.

VULGAR FRACTIONS. 37

The *Fractions* are $\frac{1}{2}$, $\frac{3}{4}$ of $\frac{2}{3}$ and $\frac{1}{4}$. Now $\frac{3}{4}$ of $\frac{2}{3}$ = $\frac{1}{2}$. Therefore add $\frac{1}{2}$, $\frac{1}{2}$ and $\frac{1}{4}$ together.

These *Fractions* reduced to a common Denominator will be $\frac{40}{80}$, $\frac{30}{80}$ and $\frac{20}{80}$. These Numerators added together, and divided by the common Denominator 80, gives 1 whole Number in the Quotient, and 408 remains over. So is the Sum of the *Fractions* only 1 $\frac{408}{80}$, which I add to the whole Numbers as follows, and have the same Answer as above.

Add 4 £.

7

1 $\frac{408}{80}$

Ans. 12 $\frac{408}{80}$ as before = 12 £. 11 s. 4d.

Tyr. This Way is the best I see; because if the whole Numbers consist of many Places of Figures, then by the first Method in reducing them to *improper Fractions*, there will be a great many Figures, and a great deal of Work to reduce them to a common Denominator afterwards; whereas, if the whole Numbers be ever so large, it makes no Alteration in this second Way of doing it.

Phi. Your Notion is right, *Tyrunculus*; and I think you will soon learn to *subtract*, you are so perfect in *Addition*.

Tyr. I will do the best I can.

Phi. Well, *Tyrunculus*, who can desire any further? You have done well hitherto, and I hope will continue it. We will proceed then to.

E S E C T.

S E C T. II

Of SUBTRACTION of VULGAR FRACTIONS.

Tyr. I Have heard that *Subtraction* is the hardest Rule in *Fractions*.

Pbi. It is counted so by most Learners; however, he that understands *Reduction* well may soon do it, as you will presently find.

Tyr. Pray how is it performed?

Pbi. The very same as *Addition*, save only you are to subtract instead of adding, according to the Nature of the Question; but the Rule is the same. For all *compound Fractions* must be reduced to *simple-ones*, and then all to a common Denominator; after which only take the Numerator of the *Fraction* to be subtracted, out of the Numerator of the other *Fraction*, and you have the Difference or Answer as in common *Subtraction*.

Ex. 1.	Ex. 2.	Ex. 3.
From $\frac{19}{24}$	From $\frac{14}{18}$	From $\frac{136}{144}$
Take $\frac{11}{24}$	Take $\frac{10}{18}$	Take $\frac{98}{144}$
Ans. $\frac{8}{24}$	Ans. $\frac{4}{18}$	Ans. $\frac{38}{144}$
Proof. $\frac{12}{24}$	Proof. $\frac{44}{18}$	Proof. $\frac{116}{144}$

Here in these three Examples, because the *Fractions* have a common Denominator, I only subtract the Numerators as in common *Subtraction*, and place the Difference over the common Denominator for an Answer. I prove the Work also as in common *Subtraction*; for I add the Numerator of the Difference to the Numerator of the less *Fraction*, which, if the Work

VULGAR FRACTIONS. 39

Work be right, will be equal to the Numerator of the greater *Fraction*.

Tyr. I think this is more diverting than *Addition*. Now let me ask you a Question or two if you please.

EXAMPLE 4.

From $\frac{11}{12}$ take $\frac{1}{7}$.

Phi. First, I reduce the *Fractions* to a common Denominator, and find them $\frac{77}{84}$ and $\frac{12}{84}$, then I subtract the Numerator 12 from the Numerator 77, and there remains 65; which placed over the common Denominator 84, gives $\frac{65}{84}$ for the Difference.

EXAMPLE 5.

From $\frac{2}{3}$ of $\frac{1}{2}$ take $\frac{2}{5}$ of $\frac{1}{3}$

That is, from $\frac{2}{3}$ take $\frac{2}{5}$. These reduced to a common Denominator, it will be, From $\frac{4}{6}$ take $\frac{2}{6}$. Now these are prepared for Work; therefore, by subtracting 2 from 4, I have 2 remaining: So is $\frac{2}{6}$ the Difference between $\frac{2}{3}$ of $\frac{1}{2}$ and $\frac{2}{5}$ of $\frac{1}{3}$.

Tyr. There can be Nothing easier than these Examples; he that understands *Addition* cannot miss *Subtraction* indeed. But still I have taken Notice of one Thing, which perhaps if I mention you will laugh at me.

Phi. Why should you think so; that would be highly base in me, when I have before desired you to ask me any Thing that you are doubtful of; therefore pray let's hear it, it may perhaps be of more Service than you are aware of?

Tyr. It is this then: In all the foregoing Examples I perceive that the Numerator of the *Fraction* to be subtracted is less than the Numerator of the *Fraction* you subtract from, which makes all the Examples

quite easy: But suppose the Numerator of the *Fraction* to be subtracted be larger than the other *Fraction*, where can I take it out of then, and how must I proceed in such a Case?

Phi. You were afraid I should laugh at you, but I assure you it is a very material Question, for this is the most difficult Part of *Subtraction* that Learners meet with. The Rule then is this:

N. B. When the Numerator of the lower *Fraction* (that is, the *Fraction* to be subtracted) is larger than the *Fraction* you subtract from, then take the said Numerator out of the common Denominator, and to that Difference add the top, or less Numerator, so shall this be a new Numerator to be placed over the common Denominator, and you must carry one for borrowing out of the common Denominator, as you do when you borrow in common *Subtraction*, when the lower Figure is larger than the Top-one.

Tyr. This is quite plain, and easy enough to be performed I should think.

Phi. Easy; can a Thing be hard, when the Rule laid down to work it by tells you how to proceed in every Respect? However, I will try you with a Question.

EXAMPLE 6.

From $24 \text{ } \frac{5}{10}$, take $19 \text{ } \frac{6}{10}$.

Tyr. I try to subtract the Numerator 6 from the Numerator 5, but cannot; therefore by the Rule I take 6 out of the common Denominator 10, and there remains 4, to which I add the less Numerator 5, and that makes 9, which I place over the common Denominator 10, and it is $\frac{9}{10}$. Then because I borrowed out of the common Denomination 10, I carry 1 to the whole Number 19, and it makes 20, which

VULGAR FRACTIONS. 41

which I subtract from 24, and there remains 4. So is the Difference 4 $\text{L. } \frac{2}{10}$, as under.

EXAMP. 7.

$$\begin{array}{r} \text{L.} \\ \text{From } 24 \frac{3}{10} \\ \text{Take } 19 \frac{6}{10} \\ \hline \end{array}$$

$$\text{Ans. } 4 \frac{2}{10}$$

Prov'd as in Ex. 1. $24 \frac{1}{10}$

$$\begin{array}{r} \text{L.} \\ \text{From } 419 \frac{2}{10} \\ \text{Take } 147 \frac{27}{10} \\ \hline \end{array}$$

$$\text{Ans. } 271 \frac{28}{10}$$

Proof $419 \frac{2}{10}$

Phi. A Proof of EXAMPLE 6. by common Subtraction.

$$\begin{array}{r} \text{L.} \quad \quad \quad \text{L. } s. \\ \text{From } 24 \frac{3}{10}, \text{ that is, } 24 \frac{10}{10}, \text{ by Case 9. of Reduction.} \\ \text{Take } 19 \frac{10}{10}, \text{ that is, } 19 \frac{12}{10} \\ \hline \end{array}$$

$$\text{Ans. } 4 \frac{2}{10}, \text{ that is, } 4 \frac{18}{10} \text{ Difference.}$$

Proof $24 \frac{3}{10}, \text{ that is, } 24 \frac{10}{10}$

Tyr. This is pretty to see the Proof of one Rule of Arithmetic by another.

Phi. You may prove any of the four Rules in Fractions by common Arithmetic as well as this; for they will come exactly alike if you proceed in a right Manner. Well, Tyrunculus, you seem to me to be qualified for Multiplication, but I have a Fancy to try you with one Question more, which will make you Master of Subtraction.

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EXAMPLE 8.

A lent B 240 £. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$; and B paid him 176 £. $\frac{7}{8}$ of $\frac{11}{12}$; what is still due to A?

Tyr. I proceed thus:

A 240 £. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$; and B 176 £. $\frac{7}{8}$ of $\frac{11}{12}$.
Now $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6} = \frac{5}{12}$; and $\frac{7}{8}$ of $\frac{11}{12} = \frac{77}{96}$. Then $\frac{5}{12}$ and $\frac{77}{96}$, reduced to a common Denominator, will be $\frac{40}{96}$ and $\frac{77}{96}$. Thus are the Fractions prepared, and will stand thus:

	£.	
A lent B	240	$\frac{40}{96}$
B paid him	176	$\frac{77}{96}$

Then by Ex. 6.

I find the Balance 63 $\frac{15}{96}$; which, by Case 9. in Reduction, is 63 £. 8s. 9d. $\frac{1}{2}$ $\frac{2208}{1000000}$.

Phi. I must needs say it is a Pleasure to me to instruct you, Tyrunculus. I have but one Thing more to observe to you, and then we will go to Multiplication.

Note, When you are to add or subtract the Fraction of a Farthing or a Penny from the Fraction of a £. or Guinea, &c. then (by Case the 8th. in Reduction) reduce the Fraction of the one, and make it equivalent to the Fraction of the other, and add or subtract as the Question requires, you have the Answer. And thus much for Subtraction.

S E C T. III.

MULTIPLICATION of VULGAR FRACTIONS.

Tyr. HOW is Multiplication of Fractions performed?

Phi. As in common Arithmetic, so also here are two Parts or Factors given, viz. the Multiplicand and Multiplier.

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Multiplier. When therefore you have reduced the mixt Numbers to *improper Fractions*, and the *compound* to *simple Fractions*, the Rule is, multiply the two Numerators together for a *new Numerator*, and the Denominators together for a *new Denominator*, and you have the Product or Answer; which, if it be an *improper Fraction*, reduce to a mixt Number, and the Work is done.

EXAMPLE I.

Multiply $\frac{4}{3}$ by $\frac{5}{8}$.

Ans. $\frac{20}{24} = \frac{5}{6}$.

EXAMPLE 2.

Multiply $\frac{42}{1}$ by $\frac{38}{24}$.

Here $42 \times 24 = 1008$ N. N. and $56 \times 38 = 2128$ N. D. *Ans.* $\frac{1008}{2128}$.

Tyr. Nothing is easier than this indeed. Pray try me with a few Questions?

Phi. I will; and I am fully persuaded that you will work most of them, if you rightly observe the Rule, which pray look at once more, lest you be not perfect in it.

EXAMPLE 3.

Multiply $\frac{4765}{9}$ by $\frac{14}{29}$.

Tyr. First $4765 \times 14 = 66710$ for N. N. and $9 \times 29 = 261$ N. D. *Ans.* $\frac{66710}{261} = 255 \frac{155}{261}$.

Phi. You are very right.

EXAMPLE 4.

Multiply $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{1}{2}$ of $\frac{5}{8}$.

Tyr. First $\frac{2}{3}$ of $\frac{3}{4} = \frac{2}{4}$; and $\frac{1}{2}$ of $\frac{5}{8} = \frac{5}{16}$; therefore I multiply $\frac{2}{4}$ by $\frac{5}{16}$, and the Answer is $\frac{10}{64}$. Now I'll ask you one, if you please.

Phi. Pray do,

Ex-

EXAMPLE 5.

Tyr. Multiply 41 £. $\frac{2}{3}$ by 12 £. $\frac{3}{4}$.

Phi. These being mixt Numbers, they are equal to *improper Fractions*, that is, $41 \frac{2}{3} = \frac{250}{3}$; and $12 \frac{3}{4} = \frac{51}{4}$; that is, $\frac{209}{4}$ and $\frac{51}{4}$. Now $209 \times 51 = 10659$; and $5 \times 4 = 20$; so that the *Answer* is $\frac{10659}{20}$, which reduced to a mixt Number, is $532 \frac{19}{20}$; that is 532 £. 19s.

Tyr. Now I think I can do any Question in this Rule.

Phi. Perhaps so; but it runs in my Head that I can puzzle you in one of the two next Examples.

EXAMPLE 6.

Multiply 14 £. $\frac{3}{4}$ of $\frac{5}{6}$, by 9 £. $\frac{3}{4}$ of $\frac{13}{18}$.

Tyr. I know I can do this. First, $\frac{3}{4}$ of $\frac{5}{6} = \frac{5}{8}$, viz. $= \frac{5}{8}$; and $\frac{3}{4}$ of $\frac{13}{18} = \frac{13}{24}$, viz. $= \frac{13}{24}$. Therefore I multiply 14 £. $\frac{5}{8}$ by 9 £. $\frac{13}{24}$. These reduced to *improper Fractions* will stand thus; multiply $\frac{117}{8}$ by $\frac{13}{2}$. Now $117 \times 19 = 2223$ N. N. and $8 \times 2 = 16$ N. D. that is, $\frac{2223}{16}$ Ans. $= 138 \frac{15}{16} = 138$ £. 18s. 9d.

Phi. Very well done; and the better because you abbreviated the *Fractions* $\frac{5}{8}$ and $\frac{13}{24}$; for which you will see the Reason given in the Method of *Abbreviations*, Dialogue 4. Sect. 2. Now for the second Question, Tyrunculus.

EXAMPLE 7.

Multiply 14 £. $\frac{5}{8}$ by 13 £.

Tyr. Let me see 14 £. $\frac{5}{8}$ by 13 £. —Why here is but one *Fraction* given.—I believe you have set me now indeed.

Phi.

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Phi. It was but this Instant that you said, you believ'd you could do any Sum in the Rule.—Why don't you make a *Fraction* of the whole Number 13?

Tyr. I must know how first.

Phi. O for Shame, don't you know, so plainly as I told you in *Dial 2. Observ. 4.* Pray don't forget your former Instructions, and then blame me; I thought this would put you to a Nonplus: However, it is the first great Error you have committed; so we will pass it by; but pray be more careful for the Future. I told you to make a *Fraction* of a whole Number, is only putting Unity under it. Thus 13 is $= \frac{13}{1}$.

Tyr. I blush to think I should be so remiss. However, pardon me, I know now easily how to perform it: For $14 \frac{5}{9} = \frac{131}{9}$; and $13 = \frac{13}{1}$; therefore by multiplying $\frac{131}{9}$ by $\frac{13}{1}$, I have $\frac{1703}{9} = 189 \frac{2}{3}$; that is, 189 *l.* 4*s.* 5*d.* $\frac{4}{3}$ or $\frac{1}{3}$ of a Farthing *Ans.*

Phi. Very right, *Tyrunculus*; and now I shall make some Observations, which, tho' already known to such as are well versed in *Vulgar Fractions*, yet, as they are not taken Notice of by any Author I am acquainted with, it may be serviceable to you and others.

NOTE I.

When one *simple Fraction* is multiplied by another, the Product will be a *simple Fraction*, therefore the Answer is less than Unity. It will be also less than Unity when one *compound Fraction* is multiplied by another, provided they be compounded of *simple Fractions*. (See *Observ. 1. Dial. 2.*) Thus in *Example 1* where $\frac{4}{5}$ is multiplied by $\frac{5}{6}$, the Answer is but $\frac{4}{6}$; that is, $\frac{2}{3}$ of a *l.* multiplied by $\frac{5}{6}$ of a *l.* is but $\frac{5}{6}$ of a *l.* or 10 Shillings.

NOTE

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NOTE 2.

Contrary to this, one *improper Fraction* multiplied by another is more than Unity. Thus if you take the *Fractions* in *Example 1.* and invert them, it will be $\frac{2}{3}$ of a £. multiplied by $\frac{3}{2}$ of a £. and the Answer will be $\frac{4}{2} = 2$ whole Integers, or 2 £. So that the Product now is just four Times more than it was before.

NOTE 3.

When a *simple Fraction* is to be multiplied by or with an *improper Fraction*, the Product will be sometime a *simple*, and sometimes an *improper Fraction*. Thus $\frac{2}{3}$ multiplied by $1\frac{1}{3}$ produces the *simple Fraction* $\frac{4}{3}$; but $\frac{2}{3}$ multiplied by $\frac{4}{3}$ gives $\frac{8}{9}$ an *improper Fraction* for the Answer.

NOTE 4.

When the Numerator of one *Fraction* is equal to the Denominator of the other, and the Denominator of it equal to the Numerator of the other, then will their Product be Unity or 1. Thus $\frac{2}{3}$ multiply'd by $\frac{3}{2}$, the Product is $\frac{4}{2} = 1$. (See *Dial 2. Observ. 3.*) So also $1\frac{1}{3}$ of a £. into $1\frac{1}{3}$ of a £. $= 1\frac{1}{3} = 1\frac{1}{3}$ £.

Tyr. These Observations are of great Service to ground any one in the right Notion of *Multiplication*. But pray is there no Method to prove this Rule?

Phi. Certainly there is; and this is the Beauty of *Arithmetic*, that it admits of the Proof of itself divers Ways by diverse Rules. For Instance, Suppose $2\frac{2}{3}$ of a £. were multiplied by $1\frac{1}{3}$ of a £. the Answer is $4\frac{2}{3} = 4\frac{2}{3}$ £.

PROOF.

PROOF.

Now $2\frac{3}{4} = 4 \text{ } \text{£}$. and $1\frac{3}{4} = 5$, and $4 \times 5 = 20 \text{ } \text{£}$.
as above, &c. &c. &c.

Tyr. I am highly oblig'd to you *Philomathes*, for your Care in giving me so many Examples. Pray is there any Thing more worthy my Notice, or necessary to be known in *Multiplication*?

Phi. Nothing: I have indeed been more particular already than I intended; therefore I shall pass directly to *Division*.

S E C T. IV.

Of DIVISION of VULGAR FRACTIONS.

Tyr. **H**OW do you divide one *Fraction* by another?

RULE I.

Phi. After having reduced all *mixt Numbers* and *compound Fractions* as before directed, to *single* or *improper Fractions*, the Rule is, Multiply the Numerator of the *Fraction* to be divided into the Denominator of the *Fraction* you divide by, and place their Product for a new Numerator; then multiply the Denominator of the *Dividend* into the Numerator of the *Divisor* for a new Denominator, which place under the new Numerator for an Answer. Or,

RULE

RULE 2.

If you invert the Divisor, that is, turn it into contrary Order, by setting the Numerator underneath, and the Denominator over it; then multiply the Numerators and Denominators together, as in *Multiplication*, and you have the same Answer as above.

EXAMPLE 1. by RULE 1.

Divide $\frac{2}{3}$ by $1\frac{1}{2}$. Ans. $\frac{4}{15}$.

Same EXAMPLE by RULE 2.

Multiply $\frac{2}{3}$ by $\frac{2}{3}$. Ans. $\frac{4}{15}$.

EXAMPLE 2. by RULE 1.

Divide $1\frac{1}{3}$ by $\frac{5}{11}$. Ans. $= 2\frac{4}{5}$.

Same EXAMPLE by RULE 2.

Multiply $1\frac{1}{3}$ by $\frac{11}{5}$. Ans. $1\frac{11}{5}$ as before.

Tyr. I like the second as well as the first Way.

Phi. Use which you please, providing you are but perfect in either. There is no Occasion for any more Examples, seeing that you have a Rule both for *simple and compound Fractions*. However, I'll give you an Example or two more by Way of Exercise.

Ex-

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EXAMPLE 3.

Divide $41 \text{ £. } \frac{3}{4}$ by $6 \text{ £. } \frac{3}{4}$; that is, divide $\frac{203}{4}$ by $\frac{21}{4}$.

Ans. $\frac{1903}{21} = 6 \text{ £. } \frac{74}{21}$; that is, $6 \text{ £. } 5 \text{ s. } 7 \text{ d. } \frac{16}{21}$.
And after the same Manner for *compound Fractions*.

Tyr. I understand you quite well. Pray is there any Thing to be observ'd in *Division*?

Phi. I shall make a few Remarks upon the Rule itself, which may be of Service.

NOTE I.

When an *improper Fraction*, having Unity for its Denominator, is to be divided by a *simple Fraction*, whose Numerator is also Unity, the Quotient will always be an *improper Fraction*, having Unity for its Denominator, and therefore consequently equal to a *whole Number*.

NOTE 2.

When the Denominators or Numerators are not Unity, the Quotient will sometimes be an *improper*, and sometimes a *simple Fraction*.

NOTE 3.

From hence it is easy to perceive, that *Division* of *Fractions* will answer the same End as common *Multiplication*: That is, a less Number may be brought into greater by this *Division*, contrary to
F common

Ex.

common *Division*, viz. Moidores, Guineas, or Pounds Sterling into Pence and Farthings; or Hundred-Weight into Pounds and Ounces, &c.

Tyr. What do you say, Pounds may be brought into Pence and Farthings by *Division*? I thought *Division* had made any Number *less*, and not *more*!

Phi. It is true, it does so in common *Division*, but it is quite contrary in *Vulgar Fractions*; for *here*, *more* is brought into *less* by *Multiplication*, and *less* into *more* by *Division*.

Tyr. I should be glad to see an Example of this Sort if you please; for I have heard some great Pretenders to *Arithmetic* say it cannot be done, 'tis contrary to Reason.

Phi. Please then to propose a Question yourself?

EXAMPLE I.

Tyr. It is required to bring 30 Moidores into Farthings by *Division* only?

Phi. And cannot you do it think you?

Tyr. ——— Why really at present I am at a Loss.

Phi. Pray be pleased to read over *Note 1.* in *Multiplication* and *Division*, for it is only for Want of being perfect in them, and truly understanding the Nature of different *Fractions*.

Observe then,

As one Part of the given Question is Farthings, and the other Moidores, I reduce (by *Case 8* in *Reduction*) a Farthing to the *Fraction* of a Moidore, which *Fraction* I make a Divisor; and the whole
Number

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Number 30 I make a *Fraction* of also for a Dividend; so will the Quotient be the true Answer. See the *Work*.

By *Case 6. of Reduction* $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{17} = \frac{1}{1368}$ of a Moidore for a Divisor; and 30 will be $\frac{30}{1368}$. Now $\frac{30}{1368} = \frac{38880}{1}$; that is, 38880 Farthings. Do you understand it?

Tyr. Yes, quite well; I could not have thought it had been so easy. But pray what does this Mark $\div$ signify?

Phi. It is the Sign of *Division*, and shews, that the Number before it is to be divided by the Number after it.

Tyr. Very Well. Pray try me with a Question of this Sort?

EXAMPLE 5.

Phi. It is required to bring 84 Guineas into Farthings by Division only.

Tyr. I find (by *Case 6. of Reduction*) a Farthing to be equal to $\frac{1}{160}$ of a Guinea. Then $84 \div \frac{1}{160} = 84672$ Farthings *Ans.*

Phi. Very right, and pray remember that the same is to be done by *Decimal Fractions*, by finding the Decimal of a Farthing at a Moidore or Guinea the Integer, and dividing the whole Number thereby you will have the same Answer.

From hence will naturally arise the self-evident Truth of

NOTE 4.

That when any *whole Number* is divided by a *simple Fraction*, the Quotient will be so much larger than the Dividend as the Divisor is less than Unity; but when a *simple Fraction* is divided by a *whole Number*, the Quotient will be so many Times less than the Dividend as the Divisor exceeds Unity. Thus 5 divided by $\frac{1}{4}$ is equal to 20; but $\frac{1}{4} \div 5 =$ but $\frac{1}{20}$.

And thus, *Tyrunculus*, having finished these four Rules with Variety of Examples, I shall now exercise you in them with some practical Questions in the *Rule of Proportion*; which, if duly observed, will make you a compleat Master of *Vulgar Fractions*.



DIALOGUE IV.

SECT. I.

The RULE of 3, in VULGAR FRACTIONS.

Tyr. I Am proud to think I am got thus far, and yet I almost dread the Questions you are going to set me.

Phi. If you be perfect in the foregoing Rules, you have no Reason to fear *this* at all; for it is nothing else but putting the others in Practice.

Tyr. Is not the Rule of 3. of *Fractions* wrought in the same Manner as the common Rule of 3. direct?

Phi. The very same, due Regard being had to the *Fractions*. There are two Methods, the second of which is (in general) the readiest and easiest; but you may take your Choice.

RULE I.

Having reduced all *compound* to *simple Fractions*, and all *mixt Numbers* to *improper Fractions*, then state your Question by making the first and third Number of one Name or Denomination; this done, *Multiply your second Number by your third, and divide by your first, and you have the Answer.* Or,

RULE 2.

Having reduced the *Fractions*, and placed the Numbers in Order as before directed, *Multiply the Denominator of your first Number into the Numerator*

F 3

of

of the second and third for a new Numerator; then multiply the Numerator of the first Fraction or Number into the Denominator of the second and third, for a new Denominator, which place under the new Numerator for an Answer.

EXAMPLE I.

If $\frac{3}{4}$ of a Yard cost $\frac{5}{8}$ of a £. what cost 25 $\frac{5}{8}$ Yards.

$$\text{If } \frac{3}{4} \text{ ————— } \frac{5}{8} \text{ ————— } 25 \frac{5}{8}$$

205

8

Now $\frac{205}{8} \times \frac{5}{8} = \frac{1025}{48}$; this $\div \frac{3}{4} = \frac{4100}{144} = 28 \text{ £. } \frac{68}{144} = 28 \text{ £. } 9 \text{ s. } 5 \text{ d. } 19 \frac{43}{144} = \frac{1}{144}$

Or, by RULE 2.

Having stated the Question thus, If $\frac{3}{4} \text{ — } \frac{5}{8} \text{ — } 205$, I multiply the Denominator of my first Number (viz 4.) into the Numerators of the second and third (viz. 5 and 205) and it gives 4100 for a N. Numerator. Then I multiply the Numerator of the first (viz. 3) into the Denominators of the second and third, (viz. 6 and 8) and it gives 144 for a N. Denominator. So is the Answer $\frac{4100}{144} = 28 \text{ £. } \frac{68}{144}$ as above.

Tyr. I think as you say this second Method is the best, if it were only because it saves the Trouble of Division.

Phi. It is the best Way in your plain easy Questions, but in some Respects the first Method is most practicable; however, either Way you see answers the

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the same End, and therefore you may take either of them, as Practice or Fancy may direct.

Tyr. I shall take Care to be perfect in both. Please to try me with a Question?

Phi. I will.

EXAMPLE 2.

If a Load of Wheat cost 7 £. $\frac{1}{4}$, what cost one Bushel?

Tyr. A Load being 40 Bushels, I state it thus:

If $\frac{40}{1}$ — £. 7 $\frac{1}{4}$ — $\frac{1}{1}$

24

182

24 Now $\frac{182}{24} \times \frac{1}{1} = \frac{182}{24}$; this $\div \frac{40}{1} = \frac{182}{960}$ of a £. which reduced, the Value of it is 3s. 9d. $\frac{1}{2}$ Ans.

SECOND WAY.

First I reduce the *mixt* Number 7 $\frac{1}{4}$ to an *improper Fraction*, and it is $\frac{29}{4}$, as above, then the Numbers will stand thus:

If $\frac{40}{1}$ — $\frac{29}{4}$ — $\frac{1}{1}$.

Now $1 \times 182 \times 1 = 182$ N. N. and $40 \times 24 \times 1 = 960$ N. D. So is the Answer $\frac{182}{960} = 3s. 9d. \frac{1}{2}$, as before.

Phi. Very well done, *Tyrunculus*?

Ex-

EXAMPLE 3.

What is the Interest of 219 £. $\frac{3}{4}$ for a Year at 5 £. $\frac{3}{4}$ per Cent.

Tyr. — Here I must crave your Assistance.

Phi. You shall have it in Words at length.

First, If $\frac{100}{1}$ — $5 \frac{3}{4}$ — $219 \frac{3}{4}$. My 2d. and 3d. Terms being *mixt Numbers*, I reduce them to *improper Fractions*, and they are $\frac{43}{8}$ and $\frac{1095}{8}$. Now $\frac{43}{8} \times \frac{1095}{8} = \frac{47115}{64}$; this $\div$ the 1st. Number $\frac{100}{1}$ gives $\frac{47115}{6400}$, which reduced to a *mixt Number* is 11 £. $\frac{1214}{1000}$, viz. 11 £. 16 s. 0 d. 3 qrs. $\frac{1440}{1000} = \frac{144}{100} = \frac{36}{25}$. Try it at Leisure by the second Method.

Tyr. I could not have thought *Vulgar Fractions* had been so useful.

Phi. Nothing more necessary than these and *Decimal Fractions*, for the ready finding the Interest or Value of any Thing, especially when the Questions are not in whole Numbers, as you will see by the following Examples.

EXAMPLE 4.

A Merchant makes an Assurance upon a Ship and Cargo (bound to a certain Port) valued at 4500 £. 15 s. and agrees to pay 16 Guineas per Cent. what comes the Premium or Charge of the Assurance to?

First, 16 Guineas being 16 £. 16 s. this in *Fraction* is $16 \frac{16}{20} = 16 \frac{4}{5}$; and 4500 £. 15 s. is $4500 \frac{3}{4}$. This and $16 \frac{4}{5}$ reduced to *improper Fractions* will be $\frac{84}{5}$ and $\frac{18003}{4}$. Then will the Number stand thus:

If $\frac{100}{1}$ — $\frac{84}{5}$ — $\frac{18003}{4}$.

Now

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Now by Rule 2. $1 \times 84 \times 18003 = 1512252$
 N. N. And $100 \times 5 \times 4 = 2000$ N. D. So is
 $\frac{1512252}{2000}$ the Answer, which reduc'd to a mixt Num-
 ber, you have $\pounds. 756 \frac{213}{2000} = 756 \pounds. 2 s. 6 d. \frac{480}{2000}$
 or $\frac{3}{2000}$.

EXAMPLE 5.

A buys of B $\pounds. 420 \frac{2}{3}$ Stock, and gives $\pounds. 95 \frac{1}{3}$ per
 Cent. what comes it to?

First, $420 \frac{2}{3} = \frac{1262}{3}$, and $95 \frac{1}{3} = \frac{479}{3}$. Then,
 If $\frac{100}{1} \frac{1262}{3} \frac{479}{3} \frac{479}{3}$.

Ans. $\frac{604498}{13000} = 402 \pounds. 19 s. 11 d. 2 qrs. \frac{103}{1300}$.

The Proof of this is worthy your Observation, Ty-
 runculus, to shew you the Beauty of Fractions, which
 I insert purely for your Satisfaction, to give you a
 just Idea of Things of this Nature.

PROV'D another WAY.

First, $\pounds. 95 \frac{1}{3}$ per Cent. wants $\pounds. 4 \frac{1}{3}$ per Cent. of
 being Cent. per Cent. therefore the Question may be
 read thus:

What come $\pounds. 420 \frac{2}{3}$ to, deducting $\pounds. 4 \frac{1}{3}$ per Cent.
 Then,

If $100 \frac{420 \frac{2}{3}}{1} \frac{4 \frac{1}{3}}{1} \frac{420 \frac{2}{3}}{1}$.

Work as in the last, and you will have $\frac{264103}{13000} =$
 $17 \pounds. 13 s. 4 d. 1 q. \frac{1103}{13000}$ Answer. Now this added
 to the foregoing Sum $402 \pounds. 19 s. 11 d. 2 qrs. \frac{103}{1300}$
 is equal to the original Stock proposed, viz. $\pounds. 420 \frac{2}{3}$
 $= 420 \pounds. 13 s. 4 d.$

Tyr. Indeed, Philomathes, this is delightful; pray
 continue your Examples. Phi.

Phi. I am as ready to do it as you are to ask; but remember *Tyrunculus*, (by Way of Digression) *We must cut our Garment according to the Cloth*: I have already given more Examples than I intended; but still, if you have any particular Question to ask me, I am ready to do any Thing that may be of Service.

Tyr. Sir, you are extremely kind to indulge me thus far; but what I have to add is this, That by what I have seen of this Rule, it must be very serviceable to tell the Nature or Proportion of Coins: Is it not?

Phi. To be sure it is, especially when the different Sorts of Exchange with some other current Money is not altogether equal to our Pound Sterling.

Tyr. I will ask you a Question then.

EXAMPLE 6.

A Merchant in Holland draws a Bill upon his Correspondent in London for 4280 Ducatoons, at 6 s. 3 d. $\frac{1}{2}$ each; what must he receive in Pounds Sterling?

If $\frac{1}{2}$ ——— 6 s. 3 d. $\frac{1}{2}$ ——— 4280.

First, bring the second Number into Pence, then multiply them by the Denominator $\frac{1}{2}$ and take in the Numerator 3, so will the Numbers be $\frac{1}{2}$ ——— $\frac{278}{1}$ and $\frac{4280}{1}$. Now, by Rule 2. $1 \times 378 \times 4280 = 1617840$ N. N. And $1 \times 5 \times 1 = 5$ N. D. So is $\frac{1617840}{5}$ the Answer in Pence, viz. 323568 = 1348 £. 4 s. 0 d.

Tyr. Pray prove the Work by whole Numbers?

Phi. That is done very easily by *Practice*, or several other Ways. But for common Understanding, I know none better than this: First, find the Value

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Value of 4280 Pieces, at 6s. 3d. (that is, at 75d.) each, and it is 321000d. Then you have got to find the Value of 4280 Pieces, at $\frac{3}{4}$ of a Penny each; therefore by multiplying 4280 by 3, and dividing by 5, you have 2568 Pence, which added to the other, will give you the Answer as follows:

$$4280 \text{ Pieces at } 75d. = 321000$$

$$\text{Ditto, at } \frac{3}{4}d. = 2568$$

$$\text{Ditto, at } 75\frac{3}{4}d. = 323568d. = 1348 \text{ £.}$$

4s. 0d. as above.

Tyr. Then I am always to multiply by the Numerator and divide by Denominator in such Cases; am I not?

Phi. I know no easier or shorter Way I assure you.

Tyr. It is easy enough indeed as you say, and I am oblig'd to you for so plain a Demonstration: Give me Leave to ask you a Question started the other Day in my Company, and I'll have done. It is this:

EXAMPLE 7. *

A poor Man dying leaves 20 Shillings to his four Sons, A, B, C, and D; to A he left $\frac{1}{2}$, to B $\frac{1}{4}$, to C $\frac{1}{3}$, and to D $\frac{1}{6}$, with a particular Charge that the Whole might faithfully be distributed among them; it is demanded what each Legacy amounts to?

Phi. I have not room to insert the whole Work, but will plainly tell you the Method of doing any Thing of this Sort.

First,

* This and the foregoing Example were added by Desire of a Friend.

First, I take the $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ of 20 s. and adding the Parts together, I find the Sum but 19 s. So that the *Executor* has 1 s. in Hand, and the Will is not fulfilled. Then say, If 19 s. give $\frac{1}{3}$, (*viz.* 6 s. 8 d.) what will 20 s. give? Proceed thus with the rest, and you will find the Answer to be as under:

A's Share	0	7	0	$\frac{4}{19}$
B's —	0	5	3	$\frac{3}{19}$
C's —	0	4	2	$\frac{2}{19}$
D's —	0	3	6	$\frac{2}{19}$

Sum 1 0 0 $\frac{0}{19}$ *Ans.*

This shews the Parts are not always equal to the Whole, but sometimes less and sometimes more; for had he left the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and the $\frac{1}{5}$ of 20 s. it would amount to 25 s. 8 d. which you see is 5 s. 8 d. more than the Whole, though each Part is separately such an equal Part of the Whole without any Remainder. Their Shares then of this 5 s. 8 d. are found as above, only now saying, If 25 s. 8 d. be 20 s. what will $\frac{1}{2}$ or 10 s. be, &c.

Note. By this same Method is found the particular Shares of every Creditor when a Man breaks or becomes a Bankrupt; as also how much in the £. his Effects amount to.

Tyr. I return you hearty Thanks, Sir, for your Care; and do assure you that I understand the Nature of what I have seen very well.

Phi. I am glad of it, *Tyrunculus*; but you will be more confirm'd in the Knowledge of *Fractions*, and will know better how to apply them to Use, if I shew you something of the Nature of *Abbreviations*.

S E C T.

S E C T. II

Of ABBREVIATIONS.

Tyr. WHAT is the Use of *Abbreviation*?

Phi. The right Understanding of *Abbreviations* will mightily help to shorten the Work, and save a Multiplicity of Figures; besides, without them it would be quite difficult to perform some Questions, at least very tedious, as you'll see by and by.

Tyr. What would you have me abbreviate the *Fractions* before I begin the Question?

Phi. To be sure I would when they will easily allow of it; for if you remember you did it yourself in *Example 6.* in *Multiplication.*

Tyr. I believe I did; however, let me have an Example or two, that I may the better understand it.

Phi. You shall.

E X A M P L E I.

If $\frac{84}{140}$ of a Load cost $\frac{70}{175}$ of a £. what cost $12 \frac{16}{25}$

First, $\frac{84}{140} = \frac{3}{5}$, $\frac{70}{175} = \frac{2}{5}$, and $\frac{16}{25} = \frac{1}{5}$. Therefore the Sum may be thus read:

If $\frac{3}{5}$ of a Load cost $\frac{2}{5}$ of a £. what cost $12 \frac{1}{5}$ Loads?

Ans. £. 8. $1\frac{1}{5} = 8$ £. 5 s. 0 d. Now is it not easier to work with $\frac{3}{5}$, $\frac{2}{5}$, and $\frac{1}{5}$, than with the original *Fractions* $\frac{84}{140}$, $\frac{70}{175}$, and $\frac{16}{25}$?

G

Tyr.

Tyr. To be sure, the Thing appears plain; and I shall take Care to observe it. Is this all you have to say upon *Abbreviations*?

Phi. No; I have something more to add, wherein you will see the Beauty and Use of *Abbreviations* yet more plainly: Besides, your Friend *Discipulus* desir'd me to communicate it to you, as very few or none have taken Notice of it in their Writings.

Tyr. Pray what is it?

Phi. It is, *To know whether a Fraction, when abbreviated, (or reduced to its lowest Terms) be equivalent in all Respects to the original, or given Fraction.*

Tyr. I know no Method of proving it, but by comparing the Value of one with the Value of the other; thus I find $\frac{2}{3}$ of a £. is equal to 13 s. 4 d. and I find $\frac{2}{3}$ of a £. to be the same; therefore I conclude that $\frac{2}{3}$ is $= \frac{2}{3}$.

Phi. It is right; but suppose you could neither abbreviate a *Fraction*, nor yet find its Value, how then would you act?

Tyr. That I do not know.

Phi. Indeed, *Tyrunculus*, though you can work the Rule of Proportion pretty well, you are no great Judge of the Nature of it I find. Observe then,

First, *As the Numerator of the Fraction, in its lowest Terms, is to its Denominator, so will the Numerator of the original or given Fraction be to its own Denominator: Or, as one Numerator to the other, so will one Denominator be to the other, &c.*

EXAMPLE 2.

To prove whether $\frac{1}{2}$ be equal to $\frac{84}{140}$, or $\frac{127}{180}$.

First, as 3 to 5, so is 84 to 140: Or, as 3 to 84, so is 5 to 140, &c. Again, As 127 to 180, so is 30480 to 43200, &c. &c. &c.

Tyr. I am yet more oblig'd to you *Philomathes*; for this must infallibly prove what you have said sure enough.

Phi. Since you are sensible of this, *Tyrunculus*, I will shew you another Way to prove it much shorter and easier than the former.

Second, When you have reduced any Fraction to its lowest Terms to prove whether it be right, multiply the Numerator of the original Fraction, by the Denominator of the abbreviated one; and the Denominator of the Original, by the Numerator of the abbreviated one; and if the Products are equal, your Work is rightly performed. Thus, Take the two Fractions above, viz: $\frac{1}{2} = \frac{2}{4}$. For $2 \times 140 = 280$ and $5 \times 84 = 280$. Again, $\frac{127}{180} = \frac{30480}{43200}$. For 30480×180 , and 43200×127 are both equal to 5486400.

Tyr. This is short and easy indeed!

Phi. I will now give you an Example or two to shew you that the Knowledge of Abbreviations are of more Use than you imagined.

EXAMPLE 3.

A and B are two Merchants, but of different Places;
 A owes B 46 £. 12 s. 9 d. Now £. 100 of A's
 current Money is equal to 140 £. of B's; what must
 A pay to remit the aforesaid Debt?

See the Work.

£.	s.	d.
46	12	9
		5

7) 233	3	9
--------	---	---

33 6 3 *Ans.*

Tyr. This is short indeed!

Phi. But you know the Reason (I hope) of my multiplying and dividing by these Figures; do you not?

Tyr. Stay, let me consider a little upon it.—I see the Reason now plain enough. They are in Proportion to each other as the current Money of the Places are, I perceive; that is, $100 : 140 :: 5 : 7$; for as 5 to 7, so is 100 to 140. Is not this the Reason?

Phi. Be sure it is; for I can multiply and divide better by 5 and 7, than by 100 and 140; besides, how much shorter is this, than to work it at Length by the *Rule of 3.* direct. So also if 108 £. of A's be equal to 45 £. of B's, then I multiply by 5, and divide by 12, because $108 : 45 :: 12 : 5$, and the Answer will be 19 £. 8 s. 7 d. $\frac{1}{2}$. Try you at it, supposing 105 £. of A's be equal to 165 £. of B's.

Tyr. I will, Sir, and I heartily thank you for this additional and useful Observation.

Phi. The Pains I have taken I shall count a Pleasure, if you make but a good Improvement: And let

let me persuade you not to meddle with *Algebra* till you are perfect in *Fractions*; for if you do, you will not be able to make a right Judgment of the *Problems*, much less know how to do them. If you think you understand what you have done, there remains but one Thing more before you enter upon *Algebra*, and that is, that you learn the *Signs* and *Characters* therein used.

Tyr. You have shewn me some already.

Phi. I know it: But there are a great many more; therefore I shall write them down, and give you them Home, that you may learn them by Heart before I see you again; which, though I am always glad of, yet, upon this Occasion, I neither desire nor expect, till you have learnt them so perfectly, as to know their Meaning the Moment you see them.

Tyr. You may depend upon it, *Philomathes*, in a short Time: Where be they?

Phi. Stay a little. — Here, *Tyrunculus*, here they are, and I wish you well to learn them.

Tyr. I heartily thank you, *Philomathes*, and am your Servant.

Phi. I am yours, *Tyrunculus*.

S E C T. III.

An EXPLANATION of the PRINCIPAL SIGNS and CHARACTERS used in ALGEBRA.

I. **T**HIS Character (+) is the Sign of *Addition*, and signifies, that the Numbers or Quantities between which it is placed, are to be added together in one Sum. Thus, $3 + 5$ shews, that 3 and 5 are to be added together. It stands for the Word *more* also. Thus, $5 + 4 + 7$, is read, 5
more

more 4, more 7, which make 16. So also, $a + b + c + d$, shews, that a , b , c , and d , are to be added together.

2. This Character (—) is the Sign of *Subtraction*, and signifies, that the Numbers or Quantities which come after it, are to be taken from the Numbers or Quantities which stand before it. Thus, $a + b - c$, shews, that the Quantity c is to be taken from the Sum of a and b . It stands for the Word *less* also. Thus, $9 - 5$, is read, 9 *less* 5, which is 4; and $b - 9$ is *b less* 9, and shews that 9 is to be taken from the Quantity b , &c.

Note farther, That (+) signifies a *positive* or *affirmative* Quantity, or *absolute* Number; but (—) signifies a *fictitious* or *negative* Quantity or Number; a Want or Deficiency. Thus -8 is 8 Times less than Nothing. So that any Number or Quantity with the Sign + being added to the same Number or Quantity with the Sign —, their Sum will be equal to Nothing. Thus 8 added to -8 is equal to (0) but -8 taken from + 8 is = 16. (See Case 2d. in *Addition and Subtraction of Algebra*.) Again,

An Asterisk (*) is frequently used for a Cypher in *Subtraction*, that is, $b + c$ taken from $b + c$, there remains * or Nothing.

2. This Character (X) is the Sign of *Multiplication*. It signifies *into*, or *multiplied by*. Thus, $4 \times 5 \times 3$, shews, that 4 is to be multiplied by or into 5, and their Product into or by 3. So also $a \times b \times c \times d$, shews the continual *Multiplication* of a , b , c , and d .

When Quantities are placed one after another, without any Sign or Character, it shews their *Multiplication*. Thus ab is $a \times b$, or a multiply'd by b . So

So $abcd$ shews the Product of a into b into c into d , &c. Joining of Quantities therefore is multiplying them together.

4. This Character ($\div$) is the Sign of *Division*, and signifies that the Numbers or Quantities before it, are to be divided by the Numbers or Quantities after it. Thus $a \div b$, shews that a is to be divided by b ; so $16 \div 4$, shews that 16 must be divided by 4.

Note, There is a better Way of expressing *Division*, and it is more frequently used; and this is by placing the Dividend a-top, and the Divisor underneath it. Thus a divided by b is set thus, $\frac{a}{b}$. So also 16 divided by 4 is thus placed, $\frac{16}{4}$, &c. &c.

5. These two Lines ($=$) are the Signs of *Equality*, and signify, that the Quantities and Numbers on the one Side of it are equal to the Numbers or Quantities on the other.

6. This Character ($\div\div$) is the Sign of *continued* or *Geometrical Proportion*. Thus, $a \div\div b \div\div c$, &c. are Quantities in *Geometrical Proportion*. But *Geometrical Proportion* is better expressed by one and the same Quantity with the Sign after them. Thus, $a, aa, aaa, aaaa, a^5, a^6, \div$ &c. are Quantities in *Geometrical Proportion*. So also 2, 4, 8, 16, 32 $\div\div$, &c. are in *continual Proportion*.

7. This Character ($:$) signifies the Word *to*; and this ($::$) signifies the Words *so is*. When they are join'd together thus, ($::::$) they are the Rule of *Proportion*; and being placed between Numbers or Quantities, (thus, $a:b::d:e$) they are thus read
or

or exprefs'd, As a to b , so is d to e . — Or thus, $4 : 6 :: 8 : 12$, is, As 4 to 6, so is 8 to 12, &c.

8. This Character (ϕ) is here used to signify *Transposition*, and shews that the Number or Quantity before which it is plac'd is in the next Line or Step transpos'd to the other Side of the Equation, with a contrary Sign.

9. This Character or Letter (Q) is also here used, and signifies *by the Question*, as you may see in the Work of the following Problems.

10. This Character ($\sqrt{}$) is a radical Sign, or Sign of the Square Root, and shews that the Number or Quantity before which it stands is to have its Square Root extracted.

11. This Character ($\sqrt[3]{}$) is the Sign of the Cube Root, and signifies the Extraction of it, as in the Square Root above.

S E C T. IV.

Of COMPOUND SIGNS or CHARACTERS.

1. THIS ($\sqrt[3]{a \times b + c}$) signifies, that the Product of a and b added to c is to have its Cube Root extracted. The same of the Square Root.

2. This, ($\sqrt{a \times b \div c - d}$) shews, that after the Quantity d is taken out of the Product of a and b more the Sum of c , the Remainder is to have its Square Root extracted.

3. This

3. This long Dash (————) is often used to link or couple Quantities together for the better reading or understanding them. Besides they are differently expressed to what they are when it is wanting. Thus, $a \times b + c + d$ has quite a different Signification to what it has with the Dash over it thus; $a \times \overline{b + c + d}$; for $a \times \overline{b + c + d}$, signifies, that the Quantity a is to be multiplied by the Sum of b , c , and d ; whereas, without the Dash, it would signify only that the Sum of the Quantities c and d is to be added to the Product of a into b . This will be best understood by Numbers.

☞ Let 6 represent a , 4 b , 8 c , and 12 d . Now $6 \times 4 + 8 + 12 = 44$; but $6 \times \overline{4 + 8 + 12}$, will be but 44. So that the Difference on Account of the Dash is 100, &c. &c. Note further,

This Dash is often join'd to the Tail of the radiant Sign, or the Sign is continued longer, which is the same; and according as how far the Dash is extended it has a different Signification.

4. Thus $(\sqrt{m + \frac{bb}{4} + dd} - c)$ signifies, that the Quantity c is to be taken out of the Square Root of $m + \frac{bb}{4} + dd$.

5. But $\sqrt{m + \frac{bb}{4} + dd} - c$ signifies, that only the Square Root of $m + \frac{bb}{4}$ is to be extracted, and then the Difference between the Quantities $dd - c$ to be added to it. The same for the Cube Root.

6. This

6. This Character ($\pm$) signifies *more or less such a Quantity*, and is used often in Extraction of Roots, compleating of Squares, &c.

7.  Figures are frequently set over Quantities, to shew how often they are expressed, and to save the Trouble of repeating or setting down the Letters so often. Thus, b^4 signifies the same as if the Quantity b was written, express'd, or set down four Times, thus, $bbbb$. So also x^8 is $xxxxxxx$, &c.





CHAP II.

DIALOGUE V.

*Between PHILOMATHES and TYRUNCULUS,
concerning ADDITION, SUBTRACTION,
MULTIPLICATION, and DIVISION of
ALGEBRA.*

Phi. ———— (*Coming to visit Tyrunculus.*)

Tyr. ———— (*Standing at his Door.*)

Phi. OOD Morrow to you *Tyrunculus.*

Tyr. Kind *Philomathes*, your
Servant; if I may be so free,
where are you walking this Morn-
ing?

Why, *Tyrunculus*, to say the Truth, I came
only for to speak to you; it is some Time ago
(if you remember) since I gave you a Paper, and I
expected before now that you would have come for
fresh Instructions and Examples; but your staying
so long began to make me think that you had given
over the Thoughts of *Algebra* again, and had neg-
lected to learn the *Signs* and *Characters* you had of
me for that Purpose.

Tyr.

Tyr. I am sorry I have given you Occasion to think so; but the Reason of my not being with you before, is because I have had some particular Business upon my Hands, and *that* you know must be minded: However, I have learnt them perfectly by Heart, and know the Meaning of them very well.

Phi. I am glad of it; and you intend, I hope, to apply them to Practice; do you not?

Tyr. To be sure I do, as far as Leisure from more material Things will allow of.

Phi. Well then, are you at Liberty to go Home with me now, I am no Ways engaged?

Tyr. If you insist upon it I will; but I had much rather you would spend an Hour or two with me now you are here; you shall be heartily welcome to such Entertainment as my little House affords, and I shall esteem it as an Instance of your Kindness. What say you?

Phi. I heartily thank you, *Tyrunculus*. Entertainment by Way of eating and drinking I regard not, any further than to satisfy the real Wants of Nature: It is the Conversation I value, and had rather please my Mind than my Appetite; therefore upon Promise that you will not put yourself to any Trouble, nor provide for me any other than *that* which you intended for yourself had I not dropt in, I will spend a few Hours with you.

Tyr. Upon Honour I will not.

Phi. Come then, *Tyrunculus*, let us be doing.

Tyr. With all Heart; and pray what is the first Thing in *Algebra* that I am to begin with?

Phi. As you understand *Vulgar Fractions*, and know the *Signs* and *Characters* you say, the very first Thing that I shew you will be *Addition*.

SECT. I.

ADDITION of ALGEBRA.

Tyr. **H**OW is *Addition of Algebra* performed?
Pbi. The same as common *Addition*, provided the Signs be both *affirmative* or both *negative*; as you will soon find by the four following Cases.

CASE I.

Of Simple Quantities or Integers, having the same Sign.

When the Quantities to be added have the same Sign, (viz. both + or both —) then add all the Co-efficients or Numbers together, (if any there be) and place the Quantity after them, with the same Sign also before them.

Ex. 1.	Ex. 2.	Ex. 3.
Add + a	+ $2b$	— $5d$
+ a	+ b	— d
+ a	+ $4b$	— $7d$
Sum + $3a$	+ $7b$	— $13d$

Ex. 4.	Ex. 5.
+ $5aadd$	— bcd
+ $4aadd$	— bcd
+ $6aadd$	— bcd
Sum + $15aadd$	— $3bcd$

Do you understand these Examples?

H

Tyr.

Tyr. Yes, except the (*b*) that stands alone in *Example 2.* and ($-d$) in *Example 3.* for they have no Number or Co-efficient before them.

Phi. It is true they have not; but they are supposed (as all single Quantities are) to have Unity or 1 placed before them, as you may see in *Example 1.* and *Example 5.* whose Sums are $3a$, and $-3bcd$.

Tyr. I understand you now very well.

Phi. You are then further to take Notice, that all Quantities that have not the *negative Sign* ($-$) placed before them, are supposed to have the *affirmative*: Or, in other Words, thus: When any Quantity has no Sign prefix'd to it, it is then an *affirmative Quantity*; thus, a is the same as $+a$, and $4b$ is $+4b$.

CASE 2.

OF SIMPLE INTEGERS or QUANTITIES,
having contrary Signs.

When the given Quantities are alike, but have unlike Signs, then subtract the one Co-efficient from the other, and place the negative or affirmative Sign to the Remainder, according to where the Excess lies; that is, if the negative Quantity has the greater Co-efficient, then the Remainder will be negative and must have the Sign ($-$); but if the affirmative be the larger, then will the Remainder have the Sign ($+$) and will be the Sum of the said Quantities.

EX. 1.

$$\begin{array}{r} 4\ bb \\ -\ 3\ bb \\ \hline +\ bb \end{array}$$

EX. 2.

$$\begin{array}{r} -\ 7\ bb \\ \quad 3\ bb \\ \hline -\ 4\ bb \end{array}$$

EX. 3.

$$\begin{array}{r} +\ 14\ aabb \\ -\ 43\ aabb \\ \hline -\ 29\ aabb \end{array}$$

EX.

Ex. 4.

$$\begin{array}{r} - \quad abcd \\ \quad 9 \quad abcd \\ \hline + 8 \quad abcd \end{array}$$

Ex. 5.

$$\begin{array}{r} 42 \quad bbdd \\ - 42 \quad bbdd \\ \hline (00) * \end{array}$$

Do you understand these Examples?

Tyr. I understand all but the 5th. for I cannot at present conceive, that $+ 42$ added to $- 42$ can be equal to Nothing, I should think rather, that subtracting them they would be equal to Nothing.

Phi. That is your Mistake, for their Difference is 84, (as you will see *Case 2.* in *Subtraction*); because the *negative* Sign makes void the *affirmative*.

Tyr. I ask Pardon, but I do not rightly apprehend it.

Phi. I think you are a little dull now. Do you not remember that I told you, (*Sec. 3. Dialogue 4.*) that this Sign ($-$) signifies a Want or Deficiency, so many Times *less* than Nothing as the Figures after it express?

Tyr. Yes I do.

Phi. Observe then, Suppose that you stood indebted to a Person 42 £. and had no Effects of any Sort to pay the Debt, then it is plain you would be 42 Times worse than Nothing; that is, have 42 Times less than a real Property of your own: Now suppose a Friend should give you 42 £. to pay off the Debt, and you do so, still it is plain you would have Nothing in Hand to begin the World again with: Consequently then $+ 42$ added to $- 42 = *$ or 0.

H 2

Tyr.

* Left the Learner should mistake the Remainder in this Example, and take it for some Quantity, I thought proper to put it in a Parenthesis, to signify it stands for a Cypher only, and not for any Quantity. See *Dial. 9. Observation 2.*

Ex.

Tyr. I am very thankful, *Philomathes*, for so plain a Demonstration.

☞ So I perceive then, that a *negative* Quantity added to an *affirmative* one, is the same as two *affirmative* Quantities subtracted from each other. Are they not?

Phi. The very same. (See *Case 2. Ex. 5. in Subtraction.*)

Tyr. Thus far then I am pretty perfect; but how must I manage when the Quantities are many in Number, and have different Signs?

☞ *Phi.* Very easily. First collect all the Quantities that have one and the same Sign into one Sum, so you will have two Quantities at last to be added as above.

Tyr. Pray add $14\ xxx - 5\ xxx + 8\ xxx - xxx - 41\ xxx + 39\ xxx$ together?

Phi. Observe then, I collect all the Quantities having one and the same Sign together, *viz.* $14\ xxx + 8\ xxx + 39\ xxx$, and these are equal to $61\ xxx$; then $- 5\ xxx - xxx - 41\ xxx = - 47\ xxx$; then $- 47\ xxx$ added to $+ 61\ xxx$, as before directed, $= 14\ xxx$ Answer, the Sum of all the Quantities. Now I will try you with a Question.

EXAMPLE 7.

Add $4\ aaa + 9\ aaa - 12\ aaa - 4\ aaa - 19\ aaa + 14\ aaa + 6\ aaa - 9\ aaa$ together.

Tyr. Nothing easier. First, $4\ aaa + 9\ aaa + 14\ aaa + 6\ aaa = + 33\ aaa$; and $- 12\ aaa - 4\ aaa - 19\ aaa - 9\ aaa = - 44\ aaa$; then $- 44\ aaa$ added to $+ 33\ aaa = - 11\ aaa$ Answer.

Phi. You are right, but still you have taken unnecessary Trouble.

Tyr. Wherein?

Phi.

Phi. Why did not I tell you that the *negative* Sign — destroys the *affirmative* +; therefore as you have 4 *aaa* + 9 *aaa*, and also — 4 *aaa* — 9 *aaa* in the Question, you needed not to have meddled with them at all, only add the rest of the Quantities, and you will find them to be + 20 *aaa* and — 31 *aaa*, whose Sum is — 11 *aaa*, as above.

Tyr. I was a little wanting indeed in this Respect.

Phi. To be sure it saves Trouble; for suppose I were to add $abb + 6abb - 9abb - abb + 4abb - 6abb + 9abb - 3abb$ together, I have only — 3 *abb* to add to + 4 *abb*, (for the rest destroy each other by contrary Signs) and their Sum is + *abb*, the Sum of all.

Tyr. I shall take Notice of it; but suppose the Quantities to be added are not alike, nor the Signs neither, how then?

Phi. You will see by the following Case.

CASE 3.

OF SIMPLE CONTRARY QUANTITIES.

When the Quantities to be added are unlike, whether they have Co-efficients or not, set them one after another, without any Alteration of the Signs, and this will be the proper Sum.

Tyr. This is easy indeed; then if I were to add $3bc + aa + g - d$ together, I imagine their Sum is the same; viz. $3bc + aa + g - d$. Is it not?

Phi. You are very right. See the following Example.

EXAMPLE I.

$$\begin{array}{r} - 5 \ bc \\ + 4 \ da \\ - \ \ xx \\ - 4 \ cg \\ \hline \end{array}$$

$$\text{Sum} - 5 \ bc + 4 \ da - xx - 4 \ cg$$

H 3

Ex-

Phi.

ADDITION of

EXAMPLE 2.

$$\begin{array}{r}
 - 57 \text{ gb} \\
 + 4 \text{ m} \\
 - \text{ggg} \\
 + 17
 \end{array}$$

$$\text{Sum } 57 \text{ gb} + 4 \text{ m} - \text{ggg} + 17$$

CASE 4.

Of COMPOUND INTEGERS or QUANTITIES.

This depends upon the three preceding Cases: For if the Quantities are alike, and have the same Sign, then add them together by Case 1, but if they have contrary Signs, collect all such together as have one and the same Sign, and subtract them from each other, setting the Sign where the Excess lies (according to Case 2.) be it + or -. But if the Quantities and Signs be both contrary, then, (by Case 3.) set them one after another, without altering any of the Signs, and you have the Total.

Ex. 1.

$$\begin{array}{r}
 \text{Add } 4x + b \\
 x + 4b \\
 \hline
 \end{array}$$

$$\text{Sum } 5x + 5b$$

Ex. 2.

$$\begin{array}{r}
 14x - 9 \\
 - 9x + 14 \\
 \hline
 \end{array}$$

$$+ 5x + 5$$

EXAMPLE 3.

$$\begin{array}{r}
 \text{Add } 4bc + x - 18 \\
 9bb - xb + 9 \\
 \hline
 \end{array}$$

$$\text{Sum } 4bc + x + 9bb - xb - 9$$

More

More EXAMPLES.

EXAMPLE 4.

$$\begin{array}{r}
 4\ abcd + 6\ bc + 9\ abd - 9 \\
 -\ abcd - 4\ bc - 5\ abd \\
 + 2\ abcd - 3\ bc + 2\ abd \\
 -\ abcd +\ bc +\ abd + 7 \\
 \hline
 \end{array}$$

Sum $4\ abcd + 7\ abd - 2$

EXAMPLE 5.

$$\begin{array}{r}
 a + 4\ b + g + ee + 7\ xx + y \\
 x + hh + nn + 4\ ff - 12\ y \\
 - a - g + 4\ ee + 4\ hh - xx - ff \\
 - 5\ ee - 3\ ff + 5\ y - nn + 5 - zz \\
 \hline
 \end{array}$$

Sum $4\ b + 6\ xx + 5\ hh + x - 6\ y + 5 - zz$

Tyr. You have made Use of all the Cases I see in these Examples.

Phi. I do it on Purpose to serve you; and do you think you are perfect in Addition? Look at Example 5 once more if you be not.

Tyr. I think I am indeed, Sir.

Phi. If so, we will pass directly to Subtraction.

S E C T. II.

SUBTRACTION of ALGEBRA.

Tyr. I Am afraid Subtraction will puzzle me, for I remember I was more set in this Rule in Vulgar Fractions than in any of the other.

Phi.

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Phi. Be not at all discouraged, for you will presently do it I am sensible, if you take Care to mind the Rules; for I shall proceed the same as in Addition.

CASE 1.

Of SIMPLE INTEGERS, having the same Sign.

When you have any Quantity given to be subtracted from another, then change the Sign of the Quantity to be subtracted into the contrary Sign; that is, if it be + make it -, and if - make it +, and then add them by the first Case of Addition, and you have the true Difference: Or, If the Quantities have one and the same Sign, then subtract the Numbers or Co-efficients as in common Subtraction, observing to place the Sign belonging to them to the Remainder.

EX. 1.	EX. 2.	EX. 3.
From 3 <i>a</i>	- 21 <i>b</i>	8 <i>bcd</i>
Take <i>a</i>	- 9 <i>b</i>	3 <i>bcd</i>
Ans. 2 <i>a</i>	- 12 <i>b</i>	5 <i>bcd</i>

EXAMPLE 4.

From - 24 <i>abd</i>
Take - 9 <i>abd</i>
Ans. - 15 <i>abd</i>

CASE 2.

Of SIMPLE INTEGERS, having contrary Signs.

When the Quantities to be subtracted have the negative Sign, and the other the affirmative, then add them together as in common Addition, and you have their true

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true Difference; to which place the affirmative Sign; but if the Quantity to be subtracted have the affirmative Sign, and the top Quantity the negative, you are to add as before, but then place the negative Sign to their Sum, and it is the true Difference.

Ex. 1.	Ex. 2.	Ex. 3.
From 4 bb	— 7 bb	14 $aabb$
Take — 3 bb	+ 3 bb	— 43 $aabb$
Diff. + 7 bb	— 10 bb	57 $aabb$

Ex. 4.	Ex. 5.
From — $abcd$	+ 42 $bbdd$
Take 9 $abcd$	— 42 $bbdd$
Difference — 10 $abcd$	+ 84 $bbdd$

EXAMPLE 5. varied.

From 42 $bbdd$	+ 42 $aabb$	— 42 $bbdd$
Take 42 $bbdd$	— 42 $aabb$	+ 42 $bbdd$
Diff. (0)	+ 84 $aabb$	— 84 $bbdd$ *

CASE 3.

Of COMPOUND QUANTITIES.

There is no Difference in the Work of these and simple Quantities, only observe to place the Sign between the Difference according as is required.

Ex. 1.	Ex. 2.
From $a + b$	$a - b$
Take $a - b + 9$	— $a + b - 9$
Ans. + $2b - 9$	$2a - 2b + 9$

Ex-

* See Case 2d, Ex. 5. in Addition.

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EXAMPLE 3.

$$\begin{array}{r} \text{From} \quad 7 \text{ aa} + b - 7 \text{ cc} + 14 \\ \text{Take} \quad - \quad \text{aa} - 4 b + 2 \text{ cc} - 10 \\ \hline \end{array}$$

$$\text{Ans.} + 8 \text{ aa} + 5 b - 9 \text{ cc} + 24$$

Tyr. Mighty pretty, and withal very easy. Pray how are unlike Quantities subtracted?

Phi. I will tell you directly.

CASE 4.

Of CONTRARY or UNLIKE QUANTITIES.

When you are to subtract unlike Quantities, let them be either simple or compound, you have no more to do, but only to set the Quantities to be subtracted directly after the Quantity you subtract from, first observing to change or alter their Signs; thus have you their true Difference.

EXAMPLE 1.

$$\begin{array}{r} \text{From} \quad a + b - 5 c \\ \text{Take} \quad g - d + 9, \text{ by changing the Signs.} \\ \hline \end{array}$$

$$\text{Ans.} a + b - 5 c - g + d - 9$$

EXAMPLE 2.

$$\begin{array}{r} \text{From} \quad 42 bc + 24 d - gg \\ \text{Take} \quad 17 b - 14 xx - bb - 15 \\ \hline \end{array}$$

$$\text{Ans.} 42 bc + 24 d - gg - 17 b + 14 xx + bb + 15$$

Tyr. Quite plain indeed: Pray what is next?

Phi. Now, *Tyrunculus*, we are come to *Multipli-*
cation.

SECT.

SECT. III.

MULTIPLICATION of ALGEBRA.

Tyr. I Think that *Subtraction* is easier than I thought for.

Phi. Nothing like Delight and Application, *Tyrunculus*, these make Things easy.

Tyr. It is very true ; but I am afraid of *Multiplication*.

Phi. Most Learners dread a fresh Rule, though they wish too to be trying at it : But what should be the Reason for such Fear, since the same Care will conquer one as well as the other.

Tyr. Pray how is *Multiplication* performed ?

Phi. There are three Things to be observed in this Rule, viz.

1. When the Quantities have the same Sign.
2. When they have contrary Signs.
3. When they have Co-efficients or Factors.

In all which Cases you are carefully to observe, that when the Signs are both *affirmative*, or both *negative*, the Product will be *affirmative* ; but when one is *affirmative*, and the other *negative*, the Product will be *negative* : For $+\times +$, or $-\times -$, produce $+$; but $+\times -$, or $-\times +$, produce less.

CASE I.

Of QUANTITIES having the same Sign.

If the Quantities have no Factors or Co-efficients, then only join the Letters representing such Quantities, one by the Side of the other, and place the Sign $+$ before

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fore them; but if they have Factors, multiply them as in common Multiplication, placing the Product before the Quantities.

Ex. 1.	Ex. 2.	Ex. 3.
Mult. a	$4\ b$	$-\ ab$
by b	c	$-\ d$
Product ba	$4\ bc$	$+ abd$

Ex. 4.	Ex. 5.
Mult. $- 6\ x$	$14\ bb$
by $- 2\ g$	$2\ b$
Product $+ 12\ xg$	$28\ bbb$

Tyr. This is easy enough indeed.

Phi. Nothing easier, therefore we will pass to Case 2.

CASE 2.

To MULTIPLY contrary Signs.

Ex. 1.	Ex. 2.	Ex. 3.
Mult. b	ba	$- 4\ bcd$
by $- a$	$- b$	$+ 6\ b$
Product $- ab$	$- bba$	$- 24\ bbcd$

EXAMPLE 4.

Multiply	$8\ xxb$
by	$- 4\ d$
Product	$- 32\ xxbd$

Tyr.

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Tyr. There needs no more Examples. Pray what is the next?

Phi. CASE 3.

Of COMPOUND QUANTITIES.

Multiply every particular Quantity in the Multiplier, by each Member or Part of the Multiplier, as you do in common Multiplication; then add them together by the Rules of Addition, and the Work is done.

Ex. 1.

$$\begin{array}{r} \text{Mult. } ab + cd \\ \text{by } d \\ \hline \end{array}$$

Product $abd + dc$

Ex. 2.

$$\begin{array}{r} 4b - cd \\ 4b \\ \hline \end{array}$$

Product $16bb - 4cdb$

EXAMPLE 3.

$$\begin{array}{r} \text{Mult. } 9b - 8 + 5x \\ \text{by } 3d \\ \hline \end{array}$$

Product $- 27bd + 24d - 15xd$

More EXAMPLES of COMPOUND QUANTITIES.

EXAMPLE 4.

$$\begin{array}{r} \text{Mult. } a + b \\ \text{by } a + b \\ \hline \end{array}$$

$$\begin{array}{r} aa + ab \\ + ab + bb \\ \hline \end{array}$$

Prod. $aa + 2ab + bb$

EXAMPLE 5.

$$\begin{array}{r} a - b \\ a - b \\ \hline \end{array}$$

$$\begin{array}{r} aa - ab \\ - ab + bb \\ \hline \end{array}$$

$aa - 2ab + bb$

I

Ex-

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EXAMPLE 6.

$$\begin{array}{r} \text{Mult. } a + b \\ \text{by } a - b \end{array}$$

$$\begin{array}{r} aa + ab \\ - ab - bb \end{array}$$

$$\text{Product } aa \quad - bb$$

EXAMPLE 7.

$$\begin{array}{r} \text{Mult. } ab + cd - 6 \\ \text{by } c + d \end{array}$$

$$\begin{array}{r} abc + ccd - 6c \\ + abd + cdd - 6d \end{array}$$

$$\text{Prod. } abc + ccd - 6c + abd + cdd - 6d$$

EXAMPLE 8.

$$\begin{array}{r} \text{Mult. } 2b + cd - 4 \\ \text{by } -7x + 3 \end{array}$$

$$\begin{array}{r} -14bx - 7cdx + 28x \\ + 6b + 3cd - 12 \end{array}$$

$$\text{Prod. } -14bx - 7cdx + 28x + 6b + 3cd - 12$$

Ex

EXAMPLE 9.

Mult. $a + b + c$
by $a + b + c$

$$\begin{array}{r} aa + ab + ac \\ + ab + bb + bc \\ + ac + bc + cc \\ \hline \end{array}$$

Prod. $aa + 2ab + 2ac + 2bc + bb + cc$

EXAMPLE 10.

Mult. $a + b + c$
by $a - b - c$

$$\begin{array}{r} aa + ab + ac \\ - ab - bb - bc \\ - ac - bc - cc \\ \hline \end{array}$$

Product $aa - bb - 2bc - cc$

Tyr. I understand the Examples very well ; I see in this last Example, that $+ab = ab$, $+ac$ and $-ac$ destroy each other when you come to add them together.

Phi. Very well observed, *Tyrunculus* ; this gives me Reason to hope you are perfect in what you have seen done.

Tyr. I hope so. But pray, *Philomathes*, let me ask a Question. I am not yet fully satisfied why $+ \times -$ should produce $-$, and that $- \times -$ should produce $+$; have you no other Way to demonstrate it but barely telling me so ?

Phi. To be sure, it is easily proved as follows. Suppose I take 4 Quantities, and make them equal to
I 2 any

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any 4 Numbers, viz. Suppose $b = 12$, $c = 4$, $d = 8$, and $f = 6$. Then place them in some such like Order as follows :

$$\text{Subtract } \left\{ \begin{array}{l} b = 12 \\ c = 4 \end{array} \right. \text{ and } \left\{ \begin{array}{l} d = 8 \\ f = 6 \end{array} \right. \text{ Subtract}$$

$$\text{Then } b - c = 8, \text{ and } d - f = 2$$

Now $b - c \times d - f = 8 \times 2 = 16$, the Product of their correspondent Numbers 8 and 2. Therefore $b - c \times d - f$ will be $= db - dc - fb + fc$, viz. $= 16$, if the Work be right.

PROOF.

$$\begin{array}{r} \text{First, } b \times d = 12 \times 8 = 96 \\ \text{And } c \times f = 4 \times 6 = 24 \end{array} \left. \vphantom{\begin{array}{r} b \times d \\ c \times f \end{array}} \right\} \text{Add}$$

$$\text{Therefore } bd + cf = 120$$

Again,

$$\begin{array}{r} d \times c = 8 \times 4 = 32 \\ f \times b = 6 \times 12 = 72 \end{array} \left. \vphantom{\begin{array}{r} d \times c \\ f \times b \end{array}} \right\} \text{Add}$$

That is, $dc + fb = 104$, which taken from 120, leaves 16; that is, $bd + cf - dc - fb = 120 - 104 = 16$, as above. Consequently therefore $+ \times -$ produces $-$, and $+ \times +$, or $- \times -$ produces $+$, Q. E. D.

Tyr. I cannot say I understand it by looking it over so quick, but I will consider of it another Time. In the mean Time, pray tell me what is the Meaning of Q. E. D.?

Pbi. They are often used in the *Mathematicks*, and set at the End of a Demonstration or Proof of any Problem, and signify, that the Thing given is not

not only done, but demonstrated and proved, and is thus read, Q. E. D. *which was to be proved.*

Tyr. I heartily thank you. Pray have you any Thing more to shew me in this Rule?

Phi. Nothing more, only leave a Sum for you to work at your Leisure, which I would have you try at.

EXAMPLE II.

$$\begin{array}{r} \text{Mult. } 3xx + 4bb - 2d \\ \text{by } 2xx - 3bb - d \end{array}$$

$$\text{Prod. } 6x^4 - xxbb - 7dxx - 12b^4 + 2bbd + 2dd.$$

SECT. IV.

DIVISION of ALGEBRA.

Tyr. I Fear you will find me but a dull Scholar at *Division*; for as common *Division* is harder than the other three Rules, I think of Course *Division* of *Algebra* must.

Phi. I confess it is something more difficult; but what of all this? *Labour overcomes all Things.* However, it is easier perhaps than you imagine.

Tyr. How is *Division* performed?

Phi. In *Division* are three Cases; in all which you are to observe, as in *Multiplication*, that if the Signs be alike in the *Divisor* and *Dividend*, the *Quotient* will be *affirmative*; but if unlike, the *Quotient* must have the *negative* Sign.

CASE I.

Of QUANTITIES having the same Sign.

If the Quantities have Co-efficients belonging to them, then divide the one by the other, as in common Division, and place the Quantities in the Quotient. But if the Quantities have no Co-efficients, then set the Dividend a-top, and the Divisor under it, Fraction-wise, and if you find the same Letters or Quantities in both, cancel or cast away such Letters, and the Remainder will be the true Quotient or Answer required. Thus, suppose I were to divide $\frac{dcb}{db}$ by db , I set it thus, $\frac{dcb}{db}$; and because I find db both in the Dividend and the Divisor, I expunge or cancel them, and then only c remains, which is the Quotient or Answer.

Ex. 1.

$$\begin{array}{l} \text{Divide } \frac{bbdc}{bc} \\ \text{by } \frac{bc}{bc} \end{array} \quad \text{Ans. } bd.$$

Ex. 2.

$$\begin{array}{l} \frac{abc}{da} \\ \text{Ans. } \frac{bc}{d} \end{array}$$

EXAMPLE 3.

$$\begin{array}{l} \text{Divide } \frac{xxddg}{xbga} \\ \text{by } \frac{xbga}{xbga} \end{array} \quad \text{Ans. } \frac{xd}{b}$$

Ex. 4.

$$\begin{array}{r} \text{Divide } 15 \ xd \\ \text{by } 5 \ b \\ \hline \text{Ans. } 3 \ x \end{array}$$

Ex. 5.

$$\begin{array}{r} - 64 \ bd \\ - 8 \ b \\ \hline + 8 \ d \end{array}$$

Ex. 6.

$$\begin{array}{r} - 4 \ abc \\ - 4 \ abc \\ \hline + 1 \end{array}$$

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CASE 2.

Of CONTRARY SIGNS and QUANTITIES.

Divide the Quantities and Co-efficients as before, and to the Quotient annex the Sign —, and the Work is done.

Ex. 1.

$$\begin{array}{r} \text{Divide } ab \\ \text{by } -a \\ \hline \end{array}$$

$$\text{Ans. } -b$$

$$\text{Proof } ab$$

Ex. 2.

$$\begin{array}{r} -15xb \\ +5b \\ \hline \end{array}$$

$$\text{Ans. } -3x$$

$$\text{Proof } -15xb$$

Ex. 3.

$$\begin{array}{r} 64bdc \\ -8bd \\ \hline \end{array}$$

$$\text{Ans. } 8c$$

$$\text{Proof } 64bcd$$

EXAMPLE 4.

$$\begin{array}{r} \text{Divide } 4abc \\ \text{by } -4abc \\ \hline \end{array}$$

$$\text{Ans. } -1$$

$$\text{Proof. } 4abc$$

Note. When the Sign and Quantities are quite unlike, and the Co-efficients cannot be divided, then set them over one another Fraction-wise and you have the Answer.

Ex. 5.

$$\begin{array}{r} \text{Divide } ab \\ \text{by } g \\ \hline \end{array}$$

$$\text{Ans. } \frac{ab}{g}$$

Ex. 6.

$$\begin{array}{r} 14abg \\ 5xx \\ \hline \end{array}$$

$$\frac{14abg}{5xx}$$

Ex. 7.

$$\begin{array}{r} xxbd \\ ac \\ \hline \end{array}$$

$$\frac{xxbd}{ac}$$

Ex. 8.

$$\begin{array}{r} 15xn \\ 12ag \\ \hline \end{array}$$

$$\frac{15xn}{12ag}$$

Tyr.

Tyr. I understand it very well; but how do you divide *compound Quantities*?

Phi. The same as in *Multiplication*, by going through every Member of the *Dividend* and *Divisor* according to the Order of common *Division*.

CASE 3.

Of COMPOUND QUANTITIES.

Rule 1.

Proceed in all Respects as before, due Regard being had to the Signs, and you have your Desire.

EX. 1.

Divide $ada + abcd$
by da

Ans. $a + bc$

EX. 2.

Divide $12dc - 15dx + 4ca - 5xa$
by $3d + a$

$4c - 5x$

Tyr. Let me ask you one Question?

EXAMPLE 3.

Divide $4abc - 24aabb - 32bad$
by $4ab$

Phi. Ans. $-1c + 6ab + 8d$

Tyr. Very well; but pray is not *Division* wrought sometimes at large, or do no Questions require it?

Phi. It is frequently so wrought, and I could have done the last Example so if I would.

Tyr. Pray do, it may perhaps give me a better Idea of *Division* than I have at present.

Phi. Observe ther.

2. When you have many Quantities, then proceed as in *Division of whole Numbers*, by seeing how many Times the *Divisor* is contained in the Quantities of the *Dividend*,

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Dividend, placing it in the Quotient; then multiply the Divisor by the Quotient and place it under the Dividend, and subtract it therefrom, and to the Remainder bring down the next Quantity or Quantities in the Dividend, and thus proceed till the whole Operation is performed.

EXAMPLE 3. *The long Way.*

$$-4ab) \quad 4abc - 24aabb - 32bad (-1c + 6ab + 8d \text{ Ans.} \\ +4abc$$

$$\begin{array}{r} 32 \\ 00 \end{array} \quad \begin{array}{r} -24aabb \\ -24aabb \end{array}$$

$$\begin{array}{r} -32bad \\ -32bad \end{array}$$

Tyr. I like this very well, and it appears much plainer to me than the other. Pray give me one more Example?

Phi. Suppose it were required to divide $bx + bd + cx + cd - xe - de$ by $x + d$, it will stand as follows.

EXAMPLE 4.

$$x+d) \quad bx+bd+cx+cd-xe-de \quad (b+c-e \text{ Ans.} \\ bx+bd$$

$$\begin{array}{r} 0 \\ 00 \end{array} \quad \begin{array}{r} cx+cd \\ cx+cd \end{array}$$

$$\begin{array}{r} 0 \\ 00 \end{array} \quad \begin{array}{r} -xe-de \\ -xe-de \end{array}$$

Ex-

EXAMPLE 5.

$$3x-6) 6xxxx - 96 \quad (2xxx+4xx+8x+16 \text{ Ans.}$$

$$6xxxx - 12xxx$$

$$\begin{array}{r} 0 \quad +12xxx - 96 \\ +12xxx - 24xx \\ \hline \end{array}$$

$$\begin{array}{r} 0 \quad +24xx - 96 \\ +24xx - 48x \\ \hline \end{array}$$

$$\begin{array}{r} 0 \quad +48x - 96 \\ 48x - 96 \\ \hline \end{array}$$

0

Tyr. I like this quite well indeed ; but I could not have thought that so short a *Dividend* would have produced so many *Quantities* in the *Quotient*.

Phi. That is easy to perceive ; because $-12xxx$ is not found in the *Dividend*. I change the Sign (which is subtracting it) and bring it down for a new *Dividend* or *Remainder*, and it will be $+12xxx$. I do the same also with $-24xx$ and $-48x$. See the

PROOF.

$$2xxx+4xx+8x+16$$

$$3x-6$$

$$\begin{array}{r} 6xxxx+12xxx+24xx+48x \\ -12xxx-24xx-48x-96 \\ \hline \end{array}$$

$$6xxxx-96.$$

Tyr. 'Tis right sure enough.

Phi.

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Phi. I shall give you then but one Example more.

EXAMPLE 6.

$$\begin{array}{r} 5b-9)20ab+5xxb-6ob+3obc-5bd+4ob-36a-9xx+108-54c+9d \\ \underline{20ab-36a} \end{array} \quad \begin{array}{r} 108-54c+9d \\ \underline{108-54c-12+8} \end{array} \quad \begin{array}{r} 7a \\ \underline{7a} \end{array}$$

$$\begin{array}{r} +5xxb-6ob \\ \underline{+5xxb-9xx} \end{array}$$

$$\begin{array}{r} -6ob+3obc \\ \underline{-6ob+108} \end{array}$$

$$\begin{array}{r} +3obc-5db \\ \underline{+3obc-54c} \end{array}$$

$$\begin{array}{r} -5db+4ob \\ \underline{-5db+9d} \end{array}$$

$$\begin{array}{r} +4ob-7a \\ \underline{+4ob-7a} \end{array}$$

Tyr. I do not understand this, that $-36a$ should stand under $5xxb$: Pray how do you subtract them?

Phi. I do not subtract it from $5xxb$, though it stands under it, but from $-36a$ further on in the Dividend, and so on for the rest.

Tyr. Then it Matters not whether the Quantities stand under each other I imagine; does it?

Phi. Not at all, as long as you can but find the same Quantity any where in the Dividend; and you would do well in such long Sums to make a particular Mark against such Quantities as you have taken down or done with, as you do in common Division.

Tyr. I'll remember it; but suppose there should be any Remainder (for I imagine the Quantities will not always fall out even) how then do you manage it?

Phi.

Phi. The same as in common *Division*, by placing it over the Divisor *Fraction-wise*. Thus, suppose I were to divide $xx - bb + xc$, by $x + b$; I find the Quotient to be $x - b$, and there remains xc ; which I place over the Divisor, and the Answer stands thus, $x - b \frac{xc}{x+b}$

See the Work.

$$\begin{array}{r}
 x+b) \quad xx - bb + xc \quad (x-b \frac{xc}{x+b} \text{ Anf.} \\
 \underline{xx + xb} \\
 -xb - bb + xc \\
 \underline{-xb - bb} \\
 * \quad * \quad xc
 \end{array}$$

Tyr. Have you any Thing more to offer in *Division*?

Phi. I think there is no Occasion for any more Examples.

Tyr. Pray what comes next?

Phi. According to the Order of *Arithmetic* the Rule of *Proportion* should follow, but I shall speak of this under *Dialogue 7.* and shew you first the Nature of *Algebraic Fractions*; though one would think there is no great Occasion, since I have been so particular in *Vulgar Fractions*, in which, if you are perfect, you cannot miss to understand the *Algebraic*, which are done one and the same Way, only with Letters instead of Numbers; and this can be no great Difficulty, since Numbers are only represented by such Letters, as may be seen in the following *Dialogue.*

DIA-

DIALOGUE VI.

SECT. I.

Of ALGEBRAIC FRACTIONS, and first of
REDUCTION.

CASE 1*.

To reduce a mixt Quantity to an improper ALGEBRAIC FRACTION.

MULTIPLY the whole Quantity by the Denominator of the Algebraic Fraction, and to the Product add the Numerator.

EXAMPLE 1.

Reduce $a \frac{b}{x}$ to an improper Fraction.

$$\begin{array}{r} a \frac{b}{x} \\ \hline xa + b \\ \hline x \end{array} \text{ Ans.}$$

EXAMPLE 2.

Reduce $a + b \frac{c}{d}$ to an improper Fraction.

$$\text{Ans. } a + b \frac{c}{d} + d = \frac{da + db + c}{K}$$

* Compare this with Case 1. in Vulgar Fractions.

Tyr. I have seen several Books of *Algebraic Fractions*, but I do not remember any such Examples as these: Are they necessary?

Phi. Certainly they are, and that you will see if you do but try the same by any Figures you please to make equal to the Quantities; and this will be some Help to you, and give you a Notion of an Equation.

In *Example 1.* let $a = 5$, $b = 3$, $x = 6$, then

$$a \frac{b}{x} = 5 \frac{3}{6}$$

$$\frac{xa+b}{x} = \frac{33}{6} \text{ Ans. or rather } \frac{30+3}{6} = \frac{33}{6}.$$

For $x \times a = 6 \times 5 = 30$; $+ b = 3$, $= \frac{33}{6}$. Q. E.

D. &c. &c.

Tyr. I think it is necessary indeed, as you say.

Phi. And the Beauty of it is, the very next Case proves it; for I shall take the same two Examples.

CASE 2*.

To reduce an IMPROPER FRACTION to a MIXT QUANTITY.

This is only the Reverse of the former, for you have no more to do but to divide the Numerator by the Denominator, and it is done.

EXAMPLE 1.

Reduce $\frac{xa+b}{x}$ to its equivalent mixt Numbers.

$$x) \frac{xa+b}{x} \left(a \frac{b}{x} \text{ Ans.}$$

$$\frac{xa}{xa}$$

$$0 + b$$

Ex*

* Compare this with Case 2. in *Vulgar Fractions*.

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EXAMPLE 2.

Reduce $\frac{da + db + c}{d}$ to its equivalent mixt Numbers.

$$d) \frac{da + db + c}{da} (a + b \frac{c}{d} \text{ Anf.}$$

$$\frac{0 + db}{db}$$

$$\frac{db}{db}$$

$$\frac{0 + c}{c}$$

CASE 3*.

To reduce a whole QUANTITY to an ALGEBRAIC FRACTION.

Multiply the given Quantity by any other Quantity, and place the Product for a Numerator, and the Quantity you multiplied by for a Denominator, and it is done.

EXAMPLE 1.

Reduce b to an Algebraic Fraction, having dc for its Denominator.

$$b \times dc = bdc \text{ Anf. } \frac{bdc}{dc} = b \text{ by the next Case.}$$

EXAMPLE 2.

Reduce a to an Algebraic Fraction, having $x + f$ for its Denominator.

$$\text{First } a \times x + f = ax + af \text{ Anf. } \frac{ax + af}{x + f}. \text{ Do you understand it?}$$

K

Tyr.

* Compare this with Case 3. in Vulgar Fractions.

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Tyr. I cannot miss it; you need not give any more Examples.

Phi. I shall not; but you shall see how that this Case is only the Reverse of the next. We shall take the same two Examples.

CASE 4*.

To abbreviate an ALGEBRAIC FRACTION.

Divide the Numerator by the Denominator, that is, expunge or cast away such Quantities as are found in both, and you have your Desire.

EXAMPLE 1.

Reduce $\frac{bdc}{dc}$ to its lowest Terms.

$\frac{bdc}{dc} \div (\text{divided by}) dc = b$. See Example 1. in last Case.

EXAMPLE 2.

Reduce $\frac{ax + af}{x + f}$ to its lowest Terms.

$\frac{ax + af}{x + f} \div x + f = a$. See Example 2. last Case.

So also $\frac{20\ aab}{60\ ba} + \frac{8\ bbd}{4\ bdgc} = \frac{1\ a}{3} + \frac{1\ b}{5gc}$.

As Fractions are abbreviated by Division, it is often customary to put Unity under such Abbreviations when the Denominator is cast away; that is, if the Answer be a whole Quantity, put the Figure

1 under it: Thus $a = \frac{a}{1}$, and $a + x + 4$, express'd like a Fraction, is, $\frac{a + x + 4}{1}$.

CASE

* Compare this with Case 4. in Vulgar Fractions.

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CASE 5*.

To reduce Quantities of unequal Denominators to ALGEBRAIC FRACTIONS, having a common Denominator.

Multiply all the Denominators continually for a common Denominator, and every Numerator into all the Denominators except its own, which shall be new Numerators.

Tyr. I think I can do this directly.

Phi. No Doubt of it, for it is the same as Case 7. in Reduction of Vulgar Fractions.

EXAMPLE I.

Reduce $\frac{a}{b}, \frac{c}{d},$ and $\frac{e}{f}$ to Fractions having a common Denominator.

Tyr. First then, $b \times d \times f = bdf$ for a common Denominator. Then, $a \times d \times f = adf$ N. N. Again, $c \times b \times f = cbf$ N. N. And lastly, $e \times d \times b = edb$ N. N. So are adf, cbf, edb , new Numerators to be placed over the common Denominator bdf , and will stand as follows :

$$\text{Ans. } \left. \begin{array}{l} \frac{adf}{bdf} = \frac{a}{b} \\ \frac{cbf}{bdf} = \frac{c}{d} \\ \frac{edb}{bdf} = \frac{e}{f} \end{array} \right\} \text{ by Case 4.}$$

Phi. You are very right, *Tyrunculus*, and I am glad to see you so tractable. You see therefore that

K 3

the

• Compare this with Case 7. in Vulgar Fractions.

the Order of *Algebraic Fractions* is the same as *Vulgar*.

Tyr. Yes, I perceive it, and I find your Words true now, that to understand *Vulgar Fractions* well saves a great deal of Trouble, that must unavoidably happen to those that are ignorant of them. Pray what comes next?

Phi. I have here shewn you three Cases more than are in general taken Notice of, that you might see the Relation that *Algebraic Fractions* bear to *Vulgar*. I shall therefore shew you now how to add them together.

S E C T. II.

ADDITION of VULGAR FRACTIONS.

Tyr. **A**S *Reduction of Algebraic Fractions* is like *Vulgar*, I imagine that *Addition* is done much after the same Manner also: Is it not?

Phi. The very same: You cannot miss. Come, add $\frac{b}{x}$, $\frac{c}{x}$, and $\frac{d}{x}$ together.

Tyr. Because the Quantities have a common Denominator, I only add the Numerators, viz. $b + c + d$, under which I place the common Denominator, x , and their Sum is $\frac{b + c + d}{x}$.

Phi. Very right.

EXAMPLE 2.

Add $\frac{d}{x}$, $\frac{c}{d}$, and $\frac{f}{a}$ together.

Tyr.

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Tyr. First $x \times d \times a = xda$ for a C. D. and $b \times d \times a = bda$ N. N. Again, $c \times x \times a = cxa$ N. N. And lastly, $f \times d \times x = fdx$ N. N. which I add as under:

$$\begin{array}{r} \text{N. Numerators } bda \\ cxa \\ fdx \end{array} \left. \vphantom{\begin{array}{r} bda \\ cxa \\ fdx \end{array}} \right\} \text{add}$$

$$\text{Ans. } bda + cxa + fdx$$

$$xda$$

Or if you set down all the new Numerators over their common Denominators, and abbreviate them by *Case* the 4th. you will see they are the same, and will be equivalent to the *Fraction* given. Thus,

$$\frac{bda}{xda} = \frac{b}{x} \cdot \frac{cxa}{xda} = \frac{c}{d}, \text{ and } \frac{fdx}{xda} = \frac{f}{a}.$$

Now I will ask you one, if you please, with *mixt Numbers*.

Phi. With all my Heart.

EXAMPLE 3.

Tyr. Add. $4x \frac{b}{g}$ and $cd \frac{a}{f}$ together.

Phi. The Quantities not being a like, I add them only by the Sign +, and add the *Fractions* to them also by the same Sign as they stand. Thus, their Sum is $4x \frac{b}{g} + cd \frac{a}{f}$. Or otherwise thus, $4x + cd + \frac{bf + ag}{gf}$. Or, $\frac{4xg + b}{g} + \frac{cdf + a}{f}$.

Tyr. If it were not too much Trouble *Philomathes*, I should be glad you would demonstrate this a little plainer to me.

Phi.

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Phi. That I'll do three Ways.

1. As the Quantities are unlike, I only place them one after the other, (according to *Case 3. Dialogue 5*) and their Sum is $a + 4x + \frac{b}{g} + cd + \frac{a}{f}$. Or,

2. By reducing the *mixt Numbers* to *improper Fractions*, their Sum is $\frac{4xg + b}{g} + \frac{cdf + a}{f}$. Or,

3. Reduce the *Fractions* $\frac{b}{g} \times \frac{a}{f}$ to a common Denominator, you will have $\frac{bf + ag}{gf}$ to which add the whole Quantities, and you will have $4x + cd + \frac{bf + ag}{gf}$.

But however, I will even do more than you desired, you shall see the *numerical Proof*. Let $x = 4$, then will $4x = 16$, and let the *Fraction* $\frac{b}{g}$ be $= \frac{1}{2}$. Make $c = 4$, and $d = 3$, then will $cd = 4 \times 3 = 12$, and let the *Fraction* $\frac{a}{f}$ be $= \frac{2}{3}$. Now at your Leisure add $16 \frac{1}{2}$, to $12 \frac{2}{3}$, you will have $29 \frac{5}{6} =$ the *improper Fraction* $\frac{4xg + b}{g} + \frac{cdf + a}{f}$ which is in Numbers $\frac{32 + 1}{2} + \frac{36 + 2}{3} = \frac{125}{6} = 29 \frac{5}{6}$

as before Q. E. D. Does this appear plain to you?

Tyr. You lay me under the greatest Obligations, *Philamathes*; it appears quite easy to me indeed.

Phi. I am glad of it, then you are qualified for *Subtraction*, and there is no Occasion to dwell any longer upon *Addition*.

SECT.

SECT. III.

SUBTRACTION of ALGEBRAIC FRACTIONS.

Tyr. **H**OW do you perform *Subtraction*?

Phi. By one general Rule, viz. If the *Fractions* have a common Denominator, subtract the Numerators, by placing the Sign (—) before that which is to be subtracted, and place the Difference over the common Denominator; and if they have different Denominators, reduce them (by *Case 5. Sect. 1.* of this *Dialogue*) to a common Denominator, and then subtract the Numerators as in *Subtraction of Algebraic Integers*.

EXAMPLE I.

$$\text{From } \frac{a}{b} \text{ take } \frac{d}{b}.$$

$$\text{Ans. } \frac{a-d}{b}.$$

Ex. 2.

$$\begin{array}{r} \text{From } \frac{x+b}{f+g} \\ \text{Take } \frac{3b-x}{f+g} \\ \hline \text{Difference } \frac{2x-2b}{f+g} \end{array}$$

Ex. 3.

$$\begin{array}{r} \frac{12x+c+b}{ad+g} \\ \frac{2x+b-c}{ad+g} \\ \hline \frac{10x+2c}{ad+g} \end{array}$$

Tyr. I must confess I do not apprehend these two last Examples.

Phi.

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Pbi. You do well to say so, for they are not to be understood by every Learner at first Sight. Observe then, the Numerators of *Example 1.* are $x + b$, and $3b - x$. Now (as before directed) I change the Sign of the *Fraction* to be subtracted, and then it will be $-3b + x$, which added (for this you must remember to do when the Signs are changed) to $x + b$, makes $2x - 2b$ for the Difference. And thus you must proceed with *Example 3.* always remembering that the *negative* and *affirmative* Sign before the same Quantity destroy each other, and you will find that $10x + 2c$ remains. The same is to be observed if the *Fractions* have not a common Denominator after they are once reduced to it. And thus much for *Subtraction*, provided you understand it.

Tyr. I thank you for this fresh Instruction; I am now pretty well grounded in this Rule I believe.

S E C T. IV.

MULTIPLICATION of ALGEBRAIC FRACTIONS.

Pbi. WELL, *Tyrunculus*, what think you of *Multiplication*?

Tyr. I think I can work it without shewing, if I am not mistaken.

Pbi. That is right, *Tyrunculus*, I love to see you bold and courageous in every new Undertaking: Come then,

Multiply $\frac{a}{b}$ by $\frac{c}{d}$.

Tyr.

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Tyr. If I remember, I am only to multiply the Numerators together for a new Numerator, and the Denominators for a new Denominator. Thus, $a \times c = ac$, and $b \times d = bd$: *Ans.* $\frac{ac}{bd}$

Phi. You are very right. Now multiply $\frac{4b+c}{g+a}$ by $\frac{3d}{9x}$

Tyr. First, $\frac{4b+c}{g+a} \times 3d = \frac{12bd+3cd}{g+a}$ for a N. N. and $g+a \times 9x = 9xg+9xa$ for a N. D.

So will the *Answer* be $\frac{12bd+3cd}{9xg+9xa}$

Phi. Very well done, *Tyrunculus*, indeed.

Tyr. Now, Sir, give me Leave to ask you one?

EXAMPLE 3.

Multiply $\frac{12ax+b-25x}{x}$ by $2b+6x$.

Phi. Why, *Tyrunculus*, the Multiplier being a whole Number, I make a Fraction of it, and it will be

Mult. $\frac{12ax+b-25x}{x}$ by $\frac{2b+6x}{1}$

Then multiplying the Numerators and Denominators together, I have

Ans. $\frac{24abx+2bb-50bx+72axx+6bx-150xx}{x}$ Or

rather, it is $24ab - 44bx + 72ax - 150x + \frac{2bb}{x}$
abbreviate. Do you understand it?

Tyr. Yes, very well, except the abbreviated Answer.

Phi.

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Phi. Pray look at *Case* the 4th. in *Reduction*. You see as I have x for the Denominator, I cast away or cancel x also in every Part of the Numerator; and as there is $-50bx + 6bx$, this will be $-44bx$; then cancelling x , it is $-44b$, and x at last is set under the Quantity 2 *abb Fraction-wise*, because x is not found in it.

Tyr. I heartily thank you, *Philomathes*, for this Information.

Phi. EXAMPLE 4.

$$\begin{array}{r} \text{Mult. } b \frac{c}{d} \\ \text{by } a \\ \hline dba + ca \\ \hline d \end{array} \text{ Ans.}$$

EXAMPLE 5.

$$\begin{array}{r} \text{Mult. } 4bx \frac{c}{d} \\ \text{by } \frac{5b}{9x} \\ \hline 20bbxd + 5cb \\ \hline 9dx \end{array} \text{ Ans.}$$

Reduce the *mixt* Numbers to *improper Fractions*, and proceed as before, you will have the Answers as above; which I beg you would try at your Leisure.

Tyr. I thank you, *Philomathes*; I will try it directly. Pray have you any Thing further to say of this Rule?

Phi. Nothing but this, that when it is required to multiply any *Fraction* by its Denominator; then cast away the Denominator, and the Numerator alone is the Answer. Thus, $\frac{d}{x} \times x$ gives d : For

$\frac{d}{x} \times \frac{x}{1} = \frac{dx}{1x}$ or $\frac{dx}{x} = d$. So also, $\frac{9b + 7dx}{4x + d} \times 4x + d$, gives $9b + 7dx$ *Ans.* = to the Numerator. This is evident; for suppose $\frac{1}{2}$ to be multiplied

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plied by 7, it is $\frac{3}{7} \times \frac{7}{1} = \frac{21}{7} = 3$ the Numerator of the original *Fraction* $\frac{3}{7}$, which in Fact is, only multiplying its Numerator by Unity or 1. And now, *Tyrunculus*, we will proceed to *Division*.

SECT. V.

DIVISION of ALGEBRAIC FRACTIONS.

Tyr. I Dare say I shall not be able to work *Division* without shewing.

Phi. Poh! You are now going to be dead-hearted again, and without Cause; and I had much rather find you as bold as you were in *Multiplication*. Consider, *Tyrunculus*, that every Learner may be compared to a young unexperienced Soldier; and though we will not call *Arithmetic* his Enemy, yet he has got many Skirmishes to go thro', and must not only fight, but that *valiantly* too, to overcome them; for a Field is seldom won by Cowardice: Besides, *Tyrunculus*, I have hitherto furnished you with Weapons proper for such Engagements as you have met with, and I shall take Care to provide you with others for every fresh Attack; and do you but learn to handle them well, and you need not fear but you will always overcome.

Tyr. You lay me under the highest Obligations to love and thank you, for being so careful of me. Pray then how is *Division* performed?

Phi. The same as in *Vulgar Fractions*. Multiply the Numerator of the Dividend unto the Denominator of the Divisor for a new Numerator, and the Denominator of the Dividend into the Numerator of the Divisor for a new Denominator.

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Ex.

Ex. 1.

Divide $\frac{b}{g}$ by $\frac{d}{c}$

$$\text{Ans. } \frac{bc}{gd}$$

Ex. 2.

Divide $\frac{14b}{g}$ by $\frac{2}{a}$

$$\frac{14ba}{2g} = \frac{7ba}{g}$$

Ex. 3.

Divide $\frac{8xx}{g}$ by $\frac{b}{4c}$

$$\frac{16xxc}{bg}$$

Tyr. I understand it very well ; but suppose the *Fractions* to have one and the same Denominator ?

Phi. Then cast away both the Denominators, and divide the Numerators only.

EXAMPLE. 4.

Divide $\frac{ba + bx + ca + cx}{5gg + d}$ by $\frac{b + c}{5gg + d}$. Then it will be thus, Divide $ba + bx + ca + cx$ by $b + c$ *Ans.* $(a + x)$

$$\begin{array}{r} 0 \quad bx + cx \\ bx + cx \\ \hline 0 \end{array}$$

Do you understand these Examples ?

Tyr. Yes I do quite well.

Phi. Where then is the Difficulty you so much apprehended ?

Tyr. I must confess that I was a little fearful just now.

Phi. I know it, I could see it in your Countenance.

Tyr. You have been so kind, that I must confess I cannot value you too much ; nor can I repay you but with Thanks.

Phi.

Of PROPORTION, &c. III

Phi. I do it to serve you, as I observ'd, and if you receive Benefit by my Instructions, return the Thanks elsewhere. I only desire you to be chearful and courageous, not timorous, for that will hinder you in your Pursuit.

Tyr. I will endeavour to follow your Advice in every Respect; but it is a dull Study for Learners for the first few Rules.

Phi. I own it is; but now you are past the worst: You have in a great Measure drawn aside the Mask, and as soon as you are acquainted with the Rule of *Proportion*, and understand a little of *Equations*, (which you will soon do) it will then fall quite off, and you will with Pleasure be ravished with the Beauty of its Face, and the Symmetry of its Parts.

Tyr. How long will it be before I come to the Rule of *Proportion*?

Phi. I shall shew you the Nature of it directly.

DIALOGUE VII.

SECT. I.

Of PROPORTION in general.

Tyr. WHAT do you mean by *Proportion*?

Phi. *Proportion* is the Relation, Respect, or Quality, that Numbers or Quantities bear to each other, by a certain *Ratio*, *Reason*, *Analogy*, or *Comparison*.

Tyr. Is not *Ratio* and *Proportion* then all one and the same?

Phi. They are often used and spoken of as one and the same Thing, but there is a Difference; for strictly speaking, *Ratio* is not *Proportion* itself; but shews the Difference of Numbers, by comparing one with the other; so that by it are seen the *Reason* or *Analogy* that the *Antecedent* bears to the *Consequent*, by such and such Comparison, and in Course the *Proportion* that the Numbers bear to each other.

Tyr. What do you mean by *Antecedent* and *Consequent*?

Phi. In any two Numbers or Quantities, the first Term is called the *Antecedent*, the second the *Consequent*. Thus, 4, 8, &c. 4 is the *Antecedent*, and 8 the *Consequent*; the *Ratio* is 4, because 8 is $4 + 4$, and the *Comparison* is 2, because 4 and 8 compared, one is twice the other. So also in any Series of Numbers, as 2, 6, 10, 14, 18, &c. 2 and 18 are the *Antecedent* and *Consequent*, and all the others between them are both *Antecedents* and *Consequents*.

Tyr. I understand you very well but pray how many Sorts of *Proportion* are there?

Phi. There is, 1. *Disjunct Proportion*, or the Rule of 3. direct. 2. *Arithmetical Proportion*, or Progression. 3. *Geometrical Proportion*, or Progression. 4. *Duplicate Proportion*. 5. *TriPLICATE Proportion*. 6. *Harmonical Proportion*. And, 7. *Contra-harmonical Proportion*, &c. But it will be sufficient for our present Purpose to speak of the first Three only, since the Knowledge of the others depend upon these.

S E C T. II.

Of DIRECT PROPORTION, or the RULE of 3.

Tyr. WHAT have I more to do with *Proportion*, since I can work the *Rule of 3.* very well?

Phi. That may be; and yet you may not rightly understand the Nature of it. A great many Persons deceive themselves in this, for though they can work a Question according to the Order of the Rule itself, yet they are quite ignorant of the *Relation* or *Proportion* which one Number bears to another, notwithstanding it is evidently known that one Half of the *Mathematics* depend upon it.

Tyr. I thought if I could but barely work the Rule it was enough, (till you shew'd me to the contrary in *Abbreviations*;) but since I see it is of such excellent Use, I beg you would explain the Nature of it a little plainer to me.

Phi. I intend it; but before I give a Demonstration, it will be quite necessary that you should be well acquainted with the following Observations, and then the Demonstration will appear quite plain to you; So that my Advice is, you would read them over again and again.

OBSERV. I.

Any three Numbers or Quantities being propounded, after you have found a fourth Number in Proportion, according to the Order of the *Rule of 3.* direct, then the Proof of such Work is easily

L 3

discovered;

discovered ; for if it be done right, the Proportion will always hold thus ;

As the 1^{st} : 2^{d} :: 3^{d} : 4^{th} .

Or, as the 1^{st} : 3^{d} :: 2^{d} : 4^{th} . That is, the 1^{st} bears the same Proportion to the 3^{d} , as the 2^{d} does to the 4^{th} .

OBSERV. 2.

The Product of the 1^{st} and 4^{th} is equal to the Product of the 2^{d} and 3^{d} . That is, the Product of the *Extremes* is equal to the Product of the *Means* : For the 3^{d} divided by the 1^{st} , is equal to the 4^{th} divided by the 2^{d} , &c.

OBSERV. 3.

The 1^{st} is equal to the Product of the 2^{d} and 3^{d} , divided by the 4^{th} .

OBSERV. 4.

The 2^{d} is equal to the Product of the 1^{st} and 4^{th} , divided by the 3^{d} .

OBSERV. 5.

The 3^{d} is equal to the Product of the 1^{st} and 4^{th} , divided by the 2^{d} .

OBSERV. 6.

The 4^{th} is equal to the Product of the 2^{d} and 3^{d} , divided by the 1^{st} .

Tyr. I could not have thought there had been such Harmony in the Rule of *Proportion*. But pray explain these Observations to me by some Demonstration ?

Phi. I will, both by Quantities and Numbers, which, if you mind, you cannot but understand it.

DE-

DEMONSTRATION.

Let the 4 Quantities x , b , c , and d , represent any 4 Numbers in direct Proportion, viz.

Let $x = 2$, $b = 4$, $c = 12$, and $d = 24$.

Literally.

Numerically.

Or	1	As $x : b :: c : d$	As $2 : 4 :: 12 : 24$	Obs. 1.
For	2	As $x : c :: b : d$	As $2 : 12 :: 4 : 24$	
And	3	$xd = cb$	$2 \times 24 = 4 \times 12$	Obs. 2.
Again	4	$\frac{b}{x} = \frac{d}{c}$	$4 \div 2 = 24 \div 12$	
And	5	$\frac{x}{b} = \frac{c}{d}$	$\frac{2}{4} = \frac{12}{24}$	Obs. 3.
Lastly	6	$\frac{x}{c} = \frac{b}{d}$	$\frac{2}{12} = \frac{4}{24}$	
	7	$\frac{c}{x} = \frac{d}{b}$	$\frac{12}{2} = \frac{24}{4}$	

From the four last Steps, each Term is found as follows :

Term	1	$x = \frac{bc}{d}$	$2 = \frac{48}{24}$	Obs. 3.
	2	$b = \frac{xd}{c}$	$4 = \frac{48}{12}$	Obs. 4.
	3	$c = \frac{xd}{b}$	$12 = \frac{48}{4}$	Obs. 5.
	4	$d = \frac{bc}{x}$	$24 = \frac{48}{2}$	Obs. 6.

I should think, *Tyrunculus*, the numerical Work is so plain, that you cannot help understanding the literal, since the Steps of one answer to the other.

Tyr.

Tyr. Nothing can be plainer indeed. But pray what do you mean by *Steps*? and what is their Use?

Phi. Those are the Steps that stand in the Margin of the Work, numbered 1, 2, 3, &c. Their Use is to shew the gradual Proceeding of the Operation, that you go on *gradatim*, or by Degrees; that is, Step by Step.

Tyr. Have you any Thing further to say of *direct Proportion*?

Phi. I have nothing more to add but this, that when Quantities or Numbers are in a *direct Proportion*, they are also *Proportionals* by *Alteration*, *Inversion*, *Divisjon*, *Conversion*, and *Composition*, &c. See *Euc.* 5. *Def.* 12, 13, &c.

S E C T. III.

Of ARITHMETICAL PROPORTION.

Tyr. WHAT is *Arithmetical Proportion*?

Phi. Numbers or Quantities are said to be in *Arithmetical Proportion*, or *Progression*, when they differ from one another by a certain *Ratio*, or the like Reason, Thus, 2, 6, 10, 14, 18, 22, &c. are Numbers in *Arithmetical Progression*, because they differ from one another by the like Reason, viz. by 4, which Difference is called the *Ratio*. So 1, 19, 37, 55, &c. differ from each other by the *Ratio* 18, as you may perceive; for $1 + 18 = 19$, $19 + 18 = 37$, $37 + 18 = 55$, &c. From hence will follow this Observation.

OBSERV.

OBSERV. I.

Any 3 Numbers or Quantities in *Arithmetical Proportion*, the Double of the *Mean* (or middle Number) is equal to the Sum of the *Extremes*.

Numerical Demonstration.

Let the 3 Numbers be 5, 13, and 21, whose *Ratio* or common Difference is 8; the Double of the *Mean* 13, is equal to the Sum of the *Extremes*, viz. 5 and 21.

Literal Demonstration.

Let x be put for the first Term 5, and let e represent the *Ratio*, which is 8. Then will $x + e = 13$ the *Mean*, and $x + 2e = 21$ the third Term.

PROOF.

Mean.		Extremes.
Add $\left\{ \begin{array}{l} x + e = 13 \\ x + e = 13 \end{array} \right.$	}	$\left\{ \begin{array}{l} x + 2e = 21 \\ x = 5 \end{array} \right.$ Add

M. doubled $2x + 2e = 26 = 2x + 2e = 26$ Sum of Ex.

Tyr. This is mighty plainly demonstrated indeed! But pray must I always put this Letter (e) to represent the *Ratio*?

Phi. This is at your Option; you may use any Letters, provided you put one for the Terms, and another for the *Ratio*, or Difference of the Terms.

OBSERV. 2.

Any 4 Numbers or Quantities in *Arithmetical Proportion*, either continued or discontinued and interrupted,

rupted, the Sum of the *Means* is equal to the Sum of the *Extremes*.

Numerical Demonstration.

Let the 4 Numbers in *Arithmetical Progression* be 4, 16, 28, and 40, whose *Ratio* is 12; then it is plain that $16 + 28$ the *Means*, is equal to $4 + 40$ the *Extremes*.

Litterally.

Let a represent the first Term 4, and put x for the *Ratio* 12; then will $a + x = 16$, one *Mean*; and $a + 2x = 28$, the other *Mean*, and $a + 3x = 40$, the last Term or *Extreme*.

PROOF.

Means.

$$a + x = 16$$

$$a + 2x = 28$$

Extremes.

$$a = 4$$

$$a + 3x = 40$$

$$\text{Means } 2a + 3x = 44 = \text{Extremes } a + 3x = 44.$$

N. B. It would be the same if the Numbers had been discontinued, provided the Interruption be between the 2^d and 3^d Term. Thus, suppose the 4 Numbers were 4, 16, 124, 136; then $4 + 136 = 16 + 124 = 140$. For there is the same *Ratio* between the 3^d and 4th, as there is between the 1st and 2^d, viz. 12.

Tyr. I heartily thank you, kind *Philomathes*: Have you any Thing further to add upon this?

Phi. I am not willing to leave any Thing out that may be serviceable; but I think I have said enough upon this Rule for your present Occasion. However, it may be expected I should teach you to work some

some Questions, or at least give you some Rules to work them by.

Tyr. I think that Mr. *Ward* (in his *Arithmetic*, Page 76) speaks of twenty Theorems belonging to this Rule; but he has given Examples only of two of them, and the other 18 I find in his *Algebra*, Page 186, but having no Rule for them, they are (I should think) beyond the Reach of most Learners. I should have liked he had given the Rule for finding them, though he had not done the Operation itself; because by a plain Theorem, or Rule to work by, any assiduous Learner would know how to put Things in Practice that are not very difficult; but how should he know when he has no Rule to go by nor any Tutor at Hand.

Phi. Had he given you the work of six Theorems with their Rules, you might with Ease have found out the rest; as you will discover by the six following Cases; the 2d and 5th of which will answer to his two in Page 74 of his Work.

CASE 1.

The Number of Places or Terms, and the Ratio or common Excess being given, to find the last Number.

Multiply the Number of Places less one, by the Ratio or common Excess; and to that Product add the first Number, and the Sum will be the last Number.

CASE 2.

The first and last Number (viz. the Extreme) and the Number of Terms being given, to find the Aggregate or total Sum of all the Series.

Add

Add the first and last Numbers together, and multiply the Sum by half the Number of Places, and you have the Total of all the Series added together. Or, in Case the Number of Places be odd, then add the first and last Numbers together, and multiply the Sum by the whole Number of Places, and divide that Product by 2, and you have the Aggregate or total Sum.

CASE 3.

The Extremes and Total given, to find the Number of Terms.

Add the *Extremes* together, and divide the Total by their Sum, and the Quotient will be equal to Half the Number of Places.

CASE 4.

The Total and Number of Terms given, to find the last Number.

Divide the Total by Half the Number of Places, or in Case the Terms be odd, divide double the Total by the Number of Terms, and the Quotient will be a Number; from which if you take the first Term, the Remainder will be the last Number.

CASE 5.

The Extremes and Number of Terms given, to find the Ratio or common Excess.

From the greater take the less *Extreme*, and the Remainder shall be a Dividend; then from the Number of Terms take Unity, (*viz.* 1.) and the Remainder shall be a Divisor; and the Quotient rising from them shall be the *Ratio*, or common Difference of the Terms.

CASE

Of PROPORTION.

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CASE 6.

The Extremes and common Excess given, to find the Number of Terms.

From the greater take the less *Extreme*, and divide the Remainder by the common Excess; then to the Quotient add Unity, or 1, and that Sum will be equal to the Number of Places.

Tyr. These Rules are very plain indeed, they need no Example.

Phi. Example and Precept are best together, therefore I will give you an *Example* in *Case* the 2d. and *Case* the 5th. and you will, no Doubt, do the rest upon first Trial.

EXAMPLE of CASE 5.

Let the Number of Places be 8, the *Extremes* 4 and 39, I demand the *Ratio*?

First, $39 - 4 = 35$ Dividend, then $8 - 1 = 7$ the Divisor, and $35 \div 7$ gives 5 the common Excess. Proof 4, 9, 14, 19, 24, 29, 34, 39.

Or literally thus:

Let $x = 4$ less *Extreme*, and $e = 39$ the greater, and $b =$ the Number of Terms; then will $\frac{e - x}{b - 1} = 5$ the *Ratio* as above.

Tyr. I understand it very well, Now give me one Example of *Case* the 2d?

Phi. I will.

M

Ex.

EXAMPLE of CASE 2.

Let the Numbers be as above, viz. 4, 9, 14, 19, 24, 29, 34, 39. It is required to find the Aggregate or total Sum of all the Terms added together.

First Number or <i>Extreme</i>	4	
last	39	
	—	
	43	Sum
This $\times \frac{1}{2}$ Number of Terms, viz.	4	
	—	
Total	172	

Or literally thus :

Let x be the first, and e the last Number, and let b represent Half the Number of Terms.

Then $x + e = 43$ as above

$$b = 4$$

Total $bx + be = 172$ as above.

So that from hence you see another Rule to find the Total, viz. Multiply the less *Extreme* and the greater separately by Half the Number of Terms, and add their Products together, it will be the Sum of all the Series. And thus, x , e , and b , may represent the *Extremes* and Half the Terms, be they ever so many, which you are carefully to observe.

Tyr. I like this very well, and I am sure it is far from being hard.

Phi. I shall leave you a Question to try at your Leisure to see if your Answer be like mine.

Three or 4 Men in Company were disputing concerning the Distance, and the Time it would take to gather up Stones laid each a Yard assunder for one
Mile

Mile in Length, and bringing each Stone back to the Place they began at. A silly bragging Jockey (who had present a good Horse) said he could ride further than contained to that in 3 Hours. A Sharper in Company taking Advantage of his Folly, said he would venture him 50 Guineas he did not ride his Horse so far in 3 Days; the Jockey unwarily consents; the Wager is staked, and he was to set out next Morning; but long before this he found it better to yield it lost than make Trial of such an Impossibility, it being 1549680 Yards = 880 Miles and a Half; which is upwards of 293 $\frac{1}{2}$ Miles a Day.

Tyr. Surprizing! I will try at it very shortly. Pray what comes next?

Phi. Geometrical Progression. = $1 \times 2 = 2$

SECT. IV.

Of GEOMETRICAL PROPOSITION.

Tyr. WHAT is Geometrical Proportion?

Phi. Geometrical Proportion, or Progression, is when Numbers or Quantities differ from each other by like Ratio or Reason, as in Arithmetical Progression, only with this Difference, that in Arithmetical Progression the Ratio is the Effect of Addition, but in this of Multiplication, by having one common Multiplier.

Tyr. Please to explain this more clearly to me?

Phi. Observe then, 2, 4, 8, 16, 32, 64, &c. are Numbers in Geometrical Proportion, and differ by double Reason the one from the other, the common Multiplier being 2. They are every one you see the Double of the preceding Number. So also 4, 12, 36,

36, 108, &c. differ by triple Reason, each Term being three Times its preceding one. And 1, 4, 16, 64, 256, &c. differ by quadruple Reason, &c. &c.

Tyr. I understand you now perfectly well.

Phi. Then you are to observe as follows.

OBSERV. I.

Any three Numbers in *Geometrical Proportion*, the Product of the *Extremes* is equal to the Square of the *Mean*; that is, equal to the middle Term multiplied by or into itself.

Let the 3 Numbers be 4, 16, and 64. Here $4 \times 64 = 16 \times 16 = 256$, &c.

Literal Demonstration.

Let x represent the first Term or *Extreme*, and let e be put for the *Ratio*, then will xe be the *Mean*, and xee the last Term, or other *Extreme*; then will $x \times xee$ be $=$ the Square of the *Mean* xe , viz. $xxee$.

PROOF.

Extremes.	Mean.
$x = 4$	$xe = 16$
$xee = 64$	$xe = 16$
Product $xxee = 256$	$xxee = 256$

Tyr. I understand the Example very well.

Phi. Once more then observe.

OBSERV.

OBSERV. 2.

Any four Numbers or Quantities in $\therefore$, either continued or interrupted (provided the Interruption be between the 2^d and 3^d Term) the Product of the *Means* is equal to the Product of the *Extremes*.

EXAMPLE

Let the 4 Numbers be 5, 15, 26, and 78 interrupted; then $5 \times 78 = 15 \times 26 = 390$. It will be easy to prove the same literally as above.

OBSERV. 3.

The *Ratio* of any Series of Numbers in $\therefore$ continued, is found only by dividing any of the Consequents by it Antecedent, that is, dividing any Number by the preceding Number.

OBSERV. 4.

When ever so many Numbers or Quantities differ by double Reason, and it is required to find the last Number of all, the general Way of most Persons is to double the 1st, 2^d, 3^d, 4th, &c. Number, and so continue to do till they have doubled as often as there are Terms given. But,

☞ There is a better Way when the Places are a great many, for you have no Occasion to double but a few of the Terms, and then multiply that Number into itself, and the Product will be the Double of the Terms wanting one; which doubled, gives the next Term, &c. &c.

Tyr. This must be further explained to me, I do not apprehend it.

Phi. It is a little dark in Words only; but you'll understand it the Moment you see it done.

Suppose then a Series of Numbers in $\div$ from 1 to 80 Places were given, which differ by double Reason, and it was required to find the last Number. First double a few of them, supposing to the 5th Place, (which may be done by the Head only) then square this Number, it shall give you the 9th Term; which doubled, gives you the 10th Term; this squared, gives the 19th Term, which multiplied by 2, gives the 20th Term; this squared gives the 39th Term; which into 2 gives the 40th; this into itself gives the 79th; and lastly, this doubled gives the 80th or last Term, &c. &c. &c.

Tyr. You need not demonstrate it any further; but how shall I find the Sum or Total of all the Series?

Phi. Very easily, by either of the following Methods.

OBSERV. 5.

To find the Sum of all the Series.

1. Multiply the last Term by the *Ratio*, or *common Excess*, and from the Product subtract the first Term; then divide the Remainder by the *Ratio* wanting 1, and it will give you the Sum of the Series. Or rather,

2. From the last Term take the first, and divide the Remainder by the *Ratio*, or *common Excess*, less Unity or 1; then multiply the Quotient by the *Ratio*, and to that Product add the first Number, and you will have the Sum of all the Series.

Tyr. I heartily thank you, kind *Philomathes*, for your Trouble.

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Tyr.

128 *Of reducing* EQUATIONS.

Tyr. I am oblig'd to you. Pray what is the next Thing we are to learn?

Pbi. You are now come to *Equations*, and pray take the greatest Care you possibly can, for the solving of *Algebraic* Questions depend upon the true Knowledge thereof.

Tyr. I will be diligent to observe what you say.

DIALOGUE VIII.

Of EQUATIONS.

SECT. I.

Of REDUCTION

Tyr. WHAT do you mean by an *Equation*?

Pbi. An *Equation* is an exact *Equality*, or the mutual Agreement of two or more Things when compared together. Thus, when a Pound *Sterling* is compared with Shillings, it is found equal to 20, and a Crown compared with Groats is equal to 15 such Pieces; therefore there can be no *Equation* where there are not two Things at least, because there can be no *Analogy* or *Comparison*: And when there are two Numbers or Quantities, or more, to be compared with each other, you will always find this Sign or Character ($=$) placed between them.

Demonstration.

Suppose x to represent a £. Sterling , and d 240 Pence its Equivalent, then it is evident that $x = d$.
Again,

Again, Suppose g to represent 5 Shillings, and e 15 Groats, then will $g = e$. But suppose g to represent a Shilling only, and e one Groat only, then there must be Numbers before the Quantities to form an Equation; for whereas before g was equal to e , now here it will be $g = 3e$, or $5g = 15e$; viz. $1s. = 3$ Groats, $5s. = 15$ Groats, &c.

Tyr. I understand the Demonstration very well.

Phi. You are further to observe, *Tyrunculus*, that in every Equation there are two Parts; that Part which stands before the Sign is called the first Part, and that after it the Second.

EXAMPLE.

Suppose $x = 4b + c$, then is x on the first Part equal to 4 Times the Quantity represented by b on the second Part, together with the Quantity represented by c added to it.

Tyr. Pray how are Equations formed?

Phi. This is a Question that I cannot answer as yet to your Understanding; but you may learn thus far, that when one or more Letters, representing any known Quantity, are found on the same Side of the Equation with other Quantities that represent unknown Quantities; then they must be so managed as to be brought on the other Side of the Equation; so that one Side of the Equation must be possessed by unknown, and the other by known Quantities, with the Sign of Equality between them; and thus will the unknown Quantity be discovered: And this is call'd *Transposition*. From hence will follow these *Axioms*, or self-evident Principles, which I beg you would get by Heart, at least so as to know their Use and Meaning.

AXIOM

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AXIOM 1.

If equal Numbers or Quantities be added to equal Numbers or Quantities, their Sum will still be equal; that is, suppose a was $= 4$, then by adding any Number or Quantity to each Side of the *Equation* (suppose 12) it will still be equal; that is, $a + 12 = 4 + 12, = 16$, &c.

AXIOM 2.

If equal Quantities or Numbers be subtracted from equal Quantities or Numbers, the Remainder will be equal. Thus, suppose $x = 12$, then by subtracting 8 from each Side, $x - 8 = 12, - 8 = 4$, &c.

AXIOM 3.

If equal Quantities be multiplied by equal Quantities, the Products will be still equal. Thus, suppose $x = 8$, and I multiply each Side by any Quantity or Number, as 12, then will $12x = 96$. This is plain from the next *Axiom*.

AXIOM 4.

If equal Quantities be divided by equal Quantities, the Quotients will be equal. Suppose $12x = 96$, then dividing by 12, x will be equal to 8, as per *Axiom 3*.

AXIOM 5.

Those Numbers or Quantities that are equal to one and the same Thing, are equal to one another; that is, suppose x , or $b - c$, or $5e + 6$, were either of them $=$ to 144, then are they also equal to each other.

You will see more of the Nature of these *Axioms* in the next *Section*, in treating of *Transposition*.

S E C T.

SECT. II.

REDUCTION by ADDITION, or, the Method of
TRANSPOSING Numbers and Quantities.

Tyr. WHAT do you mean by *Transposition*?

Phi. *Transposition* is the *transposing*, *altering*, *changing*, or *removing* any Thing from one Place to another. To *transpose* then any Number or Quantity, is only to remove it from one Side of Equation, and placing it on the other with the contrary Sign; and this answers to *Axiom* the 1st and 2^d.

Tyr. If I remember, you use this Character (ϕ) for *Transposition*: Do you not?

Phi. Yes, I shall throughout the Work, and where-ever you meet with it read the Word *transposing*.

Tyr. Very well. Please to give me some Examples in *Addition*?

Phi. I will; and pray remember, that *Addition* is nothing more than removing every *negative Quantity* to the contrary Side of the *Equation*, and making it *affirmative*.

EXAMPLE I.

Suppose $\begin{array}{l|l} 1 & x - b c. \end{array}$ Then $\phi - b$

$\begin{array}{l|l} 2 & x = c + b \end{array}$ Ans.

Or by *Axiom* 1 adding $+ b$ to each Side of the Equation, it will be

$\begin{array}{l|l} 1 & x - b = c \end{array}$

$\begin{array}{l|l} 2 & + b = + b \text{ it is.} \end{array}$

$\begin{array}{l|l} 3 & x = c + b \text{ as before; because } - b \times b \end{array}$
on the first Side of the Equation destroy each other.

Ex-

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EXAMPLE 2.

Let $\left| \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right| \begin{array}{l} x - d - b = g. \\ x - d = g + b. \\ x = g + d + b. \end{array}$ Then $\phi - b$
 And $\phi - d$
Ans.

Or by adding $+d + b$ to each Side,

$\left| \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right| \begin{array}{l} x - d - b = g \\ x + d + b = +d + b, \text{ it will be} \\ x = g + d + b \text{ as before, because } -d \\ -b \text{ and } +b \text{ on the first Side destroy each other.} \end{array}$

Thus you see *Transposition* agrees with *Axiom 1.*

Tyr. I perceive it does; but it is less Trouble to change the Signs, than it is to add equal Quantities on each Side.

Phi. It is; but still *Axioms 1.* shews you the Reason of it, which perhaps you might not have known else.

Tyr. Please to give me another Example, and prove it by Numbers?

Phi. Observe then.

EXAMPLE 3.

Let $\left| \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right| \begin{array}{l} x - bc - d = aa. \\ x - bc = aa + d. \\ x = aa + d + bc \end{array}$ Then $\phi - d$
 And $\phi - bc$
Ans.

Numerical Proof.

The Equation is $x - bc - d = aa$. Make $-bc = -12$, $-d = -8$, and $aa = 25$. Then will it be

$\left| \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right| \begin{array}{l} x - 12 - 8 = 25. \\ x - 12 = 25 + 8. \\ x = 25 + 8 + 12 = 45 = aa + d + bc \end{array}$ Then $\phi - 8$
 And $\phi - 12$
 as before.

And

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And thus you see that Quantities may be represented by any Numbers at Pleasure, and the Value of the unknown Quantity x may easily be discovered.

Tyr. I like this very well indeed. Give me some more Examples.

Phi. I will.

EXAMPLE 4.

Let $\left| \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right| \begin{array}{l} 5x - 8 = 24 - x. \text{ Then } \phi - 8 \\ 5x = 24 + 8 - x. \text{ And } \phi - x \\ 5x + x, \text{ that is, } 6x = 24 + 8 = 32 \text{ Ans.} \end{array}$

EXAMPLE 5.

Let $\left| \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right| \begin{array}{l} x - d - bc = aa - 12b. \text{ Then } \phi - bc \\ x - d = aa - 12b + bc. \text{ And } \phi - d \\ x = aa - 12b + bc + d. \text{ Lastly, } \phi - 12b \\ x + 12b = aa + bc + d. \text{ Ans.} \end{array}$

Do you understand it?

Tyr. Yes, very plainly.

Phi. Then we will proceed to *Subtraction*, in which I shall give you the same Sort of Examples as in Addition, that you may see the Nature of both the better.

SECT. III.

REDUCTION by SUBTRACTION.

Tyr. I Understand *Addition* very well, and apprehend *Subtraction* to be only the Reverse of it.

Phi. You are right, for here you have Nothing to do but to *transpose* the *affirmative* Quantities or Numbers to the other Side of the *Equation*, and place the *negative* Sign before them.

N

Ex-

EXAMPLE 1.

Let $\begin{array}{l|l} 1 & x + b = c. \text{ Then } \phi + b \\ 2 & x = c - b \text{ Ans. See Ex. 1. Addition.} \end{array}$

Or by *Axiom 2*, subtracting $-b$ from each Side, it will be the same. Thus,

$\begin{array}{l|l} 1 & x + b = c. \text{ Then subtracting} \\ 2 & -b = -b, \text{ it is} \\ 3 & x = c - b \text{ as before; for } +b - b \text{ destroy} \end{array}$
each other on the first Side.

EXAMPLE 2.

Let $\begin{array}{l|l} 1 & x + d + b = g. \text{ Then } \phi + b \\ 2 & x + d = g - b. \text{ And } \phi + d \\ 3 & x = g - b - d \text{ Ans.} \end{array}$

Or by subtracting $-d - b$ from each Side.

$\begin{array}{l|l} 1 & x + d + b = g. \text{ Then} \\ 2 & -d - b = -d - b \\ 3 & x = g - b - d, \text{ as above; because } +d \\ & +b - d - b \text{ on the first Side destroy each other.} \end{array}$

EXAMPLE 3.

Let $\begin{array}{l|l} 1 & x + bc + d = aa. \text{ Then } \phi + d \\ 2 & x + bc = aa - d. \text{ And } \phi + bc \\ 3 & x = aa - d - bc \text{ Ans.} \end{array}$

Numerical Proof.

The Equation is, $x + bc + d = aa$. Now let $+bc = 12$, $d = 8$, and $aa = 25$. Then it will be

$\begin{array}{l|l} 1 & x + 12 + 8 = 25. \text{ Then } \phi 8 \\ 2 & x + 12 = 25 - 8. \text{ And } \phi 12 \\ 3 & x = 25 - 8 - 12 = 5 \text{ Ans.} = aa - d \\ & - bc, \text{ as above.} \end{array}$

Please to compare this with *Example 3* in *Addition*, you will see the Quantities are the same, but the Difference

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Difference of the Value of x is 40 less here than it is there, because you see that what is *affirmative* there is *negative* here. And indeed I am of Opinion, that the comparing of them with each other will shew you more the Nature of each, than many Examples whose Steps are not alike, and confusedly demonstrated.

Tyr. Indeed I think it almost impossible not to understand it, so plain as you have done it.

Pbi. Here follows then

EXAMPLE 4.

Let $\begin{array}{l|l} 1 & 5x + 8 = 24 + x. \text{ Then } \phi + 8 \\ 2 & 5x = 24 - 8 + x \\ 3 & 5x - x, \text{ that is, } 4x = 24 - 8 = 16. \end{array}$
See Example 4. in Addition.

EXAMPLE 5.

Let $\begin{array}{l|l} 1 & x + d + bc = aa + 12b. \text{ Then } \phi bc \\ 2 & x + d = aa + 12b - bc. \text{ And } d \\ 3 & x = aa + 12b - bc - d. \text{ Lastly, } \phi 12b \\ 4 & x - 12b = aa - bc - d \text{ Ans. Compare} \end{array}$
this with Example 5. in Addition.

Tyr. I see the Nature of both plainly. Have you any Thing further to add?

Pbi. It may not be amiss to give you an Example to exercise you in both.

An EXAMPLE in both RULES

EXAMPLE 5.

Let $\begin{array}{l|l} 1 & 4x - g + b + c - d = 142. \text{ By } \phi - g \\ 2 & 4x + b + c - d = 142 + g. \text{ By } \phi b, \\ 3 & 4x + c - d = 142 + g - b. \text{ Then } \phi c, \\ 4 & 4x - d = 142 + g - b - c. \text{ And } \phi - d, \\ 5 & 4x = 142 + g - b - c + d \text{ Ans.} \end{array}$
N 2 Numerical

Numerical Proof.

Let $-g = -8$, $+b = 12$, $+c = 6$ and $-d = -16$, What will x be?

Then,	1	$4x - 8 + 12 + 6 - 16 = 142$.	By $\phi - 8$,
	2	$4x + 12 + 6 - 16 = 142 + 8$.	By $\phi 12$,
	3	$4x + 6 - 16 = 142 + 8 - 12$.	By $\phi 6$,
	4	$4x - 16 = 142 + 8 - 12 - 6$.	And $\phi - 16$,
	5	$4x = 142 + 8 - 12 - 6 + 16$	<i>Ans.</i> as above.
		That is,	
	6	$4x = 166 - 18$.	That is.
	7	$4x = 148$.	Therefore by dividing 148 by 4,
	8	$x = \frac{148}{4} = 37$	<i>Ans.</i>

Tyr. I like this *numerical Proof* very much, it is so plain, and the first five Steps agree so with the *literal*, that there needs no more Examples of this Sort.

Phi. I have proceeded indeed but one Step and one Quantity at a Time, because you might see the gradual Order of the Work; but you may as well *transpose* them all at one Stroke, for it is only using the contrary Sign you know; however, this is left to your Liberty and Practice.

Tyr. I understand you very well. Pray what comes next?

Phi. You are now come to *Multiplication*, where you will begin to see the Beauty of *Equations*.

S E C T. IV.

REDUCTION by MULTIPLICATION.

Tyr. **H**OW is *Multiplication of Equations* performed?

Pbi. *Multiplication* is performed as follows:

1. When there is an *Equation* between two *Fractions* having a common Denominator, then cast away or cancel the common Denominator, and the Numerators will be equal to each other.

EXAMPLE 1.

Let $\left| \begin{array}{l} 1 \\ 2 \end{array} \right| \left| \begin{array}{l} x \\ x \end{array} \right| = \frac{ab}{9} = \frac{ab}{9}$. Then
 $\left| \begin{array}{l} 1 \\ 2 \end{array} \right| \left| \begin{array}{l} x \\ x \end{array} \right| = ab$ Ans.

2. Or if the *Fractions* have not a common Denominator reduce them to one, after which expunge the common Denominator, and the new Numerators will be equal.

EXAMPLE 2.

Let $\left| \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right| \left| \begin{array}{l} x \\ \frac{10x}{50} \\ 10x \end{array} \right| = \frac{6}{10}$. These reduced to a C. D.
 $\left| \begin{array}{l} 2 \\ 3 \end{array} \right| \left| \begin{array}{l} \frac{10x}{50} \\ 10x \end{array} \right| = \frac{30}{50}$, that is,
 $\left| \begin{array}{l} 2 \\ 3 \end{array} \right| \left| \begin{array}{l} 10x \\ 10x \end{array} \right| = 30$ Ans.

3. Or if there be but one *Fraction*, and that be made equal to any whole Number or Quantity, then only multiply the whole Quantity by the Denominator of the *Fraction*, and that Product shall be equal to the Numerator.

EXAMPLE 3.

Let $\left| \begin{array}{l} 1 \\ 2 \end{array} \right| \left| \begin{array}{l} x = \frac{24a}{12} \\ 12x = 24a \end{array} \right|$, then
Ans.

4. Or, to prevent the Trouble of reducing the *Fractions* to a common Denominator, multiply the Numerator of the second *Fraction* by the Denominator of the first *Fraction*, and place the Product for a new Numerator over the second *Fraction*, so will the Numerator of the first *Fraction* be equal to it in the Second Step. Then multiply the Numerator of the first *Fraction* by the Denominator of the second *Fraction*, so will this Product be equal to the said new Numerator, and the *Equation* will be cleared from *Fractions*, and the unknown Quantity discovered.

EXAMPLE 4.

Let $\left| \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right| \left| \begin{array}{l} \frac{x}{12} = \frac{108}{4} \\ x = \frac{1296}{4} \\ 4x = 1296 \\ x = \frac{1296}{4} = 324 \end{array} \right|$ Then 108×12
 Then $x \times 4$
 Then must
Ans.

Tyr. I like this Way best I must confess.

Phi. When the Numbers or Quantities are many, it is less Trouble indeed to do it this Way. Now perhaps you will be diverted with the Proof.

Tyr. I should be glad to see the Reason indeed why $\frac{x}{12}$ is equal to $\frac{108}{4}$.

Phi.

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Phi. That you shall directly, and in a few Words. First, x is proved to be equal to 324 as above; then

$$\frac{324}{12} = \frac{108}{4} = 27. \text{ So that } \frac{x}{12} = \frac{108}{4} \text{ Q. E. D.}$$

Tyr. Very pretty indeed! But pray give me Leave to set you a Question, and to desire you to do the Work at large: I assure you it will be of great Service to me, and I shall need no more Examples of this Sort?

Phi. You know, or may know, I am always ready to serve you. Pray propose the Thing?

EXAMPLE 5.

Tyr. Suppose $\frac{3x}{4} + 12 = \frac{2x}{3} + 14$, what then is x equal to?

Phi. A very pretty useful Question, and I will do it so plain, that I believe you will be satisfied with the Manner of it, because I shall demonstrate it as a standing Rule for all such Examples. Let $\frac{3x}{4} + 12 = \frac{2x}{3} + 14$.

DEMONSTRATION.

First, I multiply the whole Number 12, and the Numerator of the second Fraction, (*viz.* $\frac{2x}{3}$) and the whole Number 14 into the Denominator of the first Fraction, *viz.* 4, and the Products are 48, $8x$ and 56: But whereas the Sign of Equality (=) falls between the whole Number 12 and the Fraction $\frac{2x}{3}$, I still keep it always in the same Place, till I have done multiplying the Whole; therefore it will be $3x$.

$+ 48 = 8x + 56$, under which I put the Denominator of the second *Fraction*, and so is the first Side of the *Equation* cleared of *Fractions*, and will stand in the second Step, thus, $3x + 48 = \frac{8x}{3} + 56$. Then to clear the second Side from *Fractions*, I now multiply every Member of the second Step into the Denominator of the second *Fraction*, viz. 3, (except it be its new Numerator $8x$) and then it will be in the third Step $9x + 144 = 8x + 168$, and thus is the whole *Equation* freed from *Fractions*. Now transposing $8x$ and $+ 144$, I have in the fourth Step $9x - 8x = 168 - 144$; that is, $x = 168 - 144 = 24$, the Value of x required. (See the Work at large as follows, and compare it with what is above.)

The Operation of EXAMPLE 5.

Let	1	$\frac{3x}{4} + 12 = \frac{2x}{3} + 14$. Then $\times 4$
	2	$3x + 48 = \frac{8x}{3} + 56$. Which $\times 3$, the Denominator of the second <i>Fraction</i> is
	3	$9x + 144 = 8x + 168$. Then $\phi 8x + 144$,
	4	$9x - 8x = 168 - 144$. That is,
	5	$x = 24$. <i>Anf.</i>

PROOF.

To prove that $\frac{3x}{4} + 12 = \frac{2x}{3} + 14$.

First $x = 24$ as above, then must $\frac{3x}{4} + 12$, that is, $\frac{72}{4} + 12 = 30$; and so also $\frac{2x}{3}$ that is $\frac{48}{3} + 14 = 30$. Q. E. D. *

Tyr.

* N. B. This is called *Synthetical Demonstration*, or *Composition*, and you may see this *Equation* turned into a *Problem*, and solved *Algebraically*, *Dialogue 10*, *Problem 29*.

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Tyr. Nothing can be plainer, nor more easy to be understood.

Pbi. If due Attention be given, as you observe, it is easy enough; I shall therefore leave one more Example with you, and hasten to *Division*.

EXAMPLE 6.

Let	1	$\frac{4x}{9} + 16 = \frac{3x}{6} - 18.$	First $\times 9$
	2	$4x + 144 = \frac{27x}{6} - 162.$	This $\times 6,$
	3	$24x + 864 = 27x - 972.$	Then $\phi 24x$ and $- 972,$
	4	$864 + 972 = 27x - 24x.$	That is,
	5	$1836 = 3x.$	Therefore
	6	$x = \frac{1836}{3} = 612$	<i>Ans.</i>

Again,

Suppose that $36 + \frac{6x}{8} = 72$, then will x be found to be $= 36$.

But here, *Tyrunculus*, you must observe, that if at any Time the *Square* or *Cube* of any unknown Quantity should in the last Step of any *Equation*, be found to be equal to such Number or Quantity, then must you extract the *Square* or *Cube Root* of such Numbers, and you will then have the Value of the unknown Quantity itself. Thus, suppose xx should at last fall out to be equal to 81, then is $x = 9$ the *Square Root* thereof; and if $xxx = 64$, then will $x = 4$ its *Cube Root*.

Ex-

EXAMPLE 7.

Let

1	$\frac{xx}{4} = 44 + 5.$	Then
2	$xx = 176 + 20,$	that is,
3	$xx = 196.$	Therefore,
4	$x = \sqrt{196} = 14$	Ans.

EXAMPLE 8.

Let

1	$\frac{xx}{b} + c + f = \frac{dg}{x}$
2	$xx + bc + bf = \frac{bdg}{x}$
3	$xxx + bcx + bfx = bdg$
4	$xxx = bdg - bcx - bfx.$
5	$x = \sqrt[3]{bdg - bcx - bfx}.$

Tyr. I need no more Examples, what you have shewn me already is sufficient.

Phi. You may indeed reduce any simple Equation by what you have seen, therefore we will proceed to *Division.*

SECT. V.

REDUCTION by DIVISION.

Tyr. **P**RAY how is *Division* of Equations performed?

Phi. When any Quantity or Quantities that are alike, possess both Sides of the Equation, divide each Side

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Side by the said Quantity, (which is the same as to reduce it to its lowest Terms) and then will one Side be still equal to the other; and if there be *Fractions*, clear the *Equation* of them, by multiplying all the Parts by the Denominators of the *Fraction*, as in *Multiplication*.

EXAMPLE 1.

Let $\left| \begin{array}{l} 1 \\ 2 \end{array} \right| \begin{array}{l} xx \pm 16x + 12x, \text{ then } \div \text{ by } x, \\ x = 16 + 12 = 28, \text{ by Axiom 4.} \end{array}$

EXAMPLE 2.

Let $\left| \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right| \begin{array}{l} xxc + bc x + dcx = cx + ff x. \text{ Then } \div c \\ xx + bx + dx = x + ff x. \text{ Then } \div x, \\ x + b + d = 1 + ff \text{ Ans.} \end{array}$

EXAMPLE 3.

Let $\left| \begin{array}{l} 1 \\ 2 \end{array} \right| \begin{array}{l} bx - cx = gg. \text{ Then } \div b - c, \\ x = \frac{gg}{b - c} \text{ Ans.} \end{array}$

Tyr. I do not rightly apprehend this last Example. Can you demonstrate it by Numbers?

Pbi. Yes to be sure, and will. Let $b = 8 - c = -4$, and $g = 12$, then $gg = 144$.

Numerical Proof of EXAMPLE 3.

Let $\left| \begin{array}{l} 1 \\ 2 \end{array} \right| \begin{array}{l} 8x - 4x = 144, \text{ then } \div \text{ by } 8 - 4 \\ x = \frac{144}{8 - 4} = \frac{gg}{b - c} = 36 \text{ Ans.} \end{array}$

Tyr.

Tyr. I heartily thank you, *Philomathes*, and am mightily pleased with it. But if there be *Fractions*, what do you say I am to do as I did in *Multiplication*?

Pbi. Yes, after having first abbreviated the Numerators, or dividing them by like Quantities or Numbers, (for the Denominators are never divided) you proceed then to multiply cross-ways, as in *Division of Fractions*, till you discover the unknown Quantity.

Tyr. Please to give me an Example?

Pbi. I will.

EXAMPLE 4.

Let	1	$\frac{84x}{x-4} = \frac{70x}{x-6}$	Then $\div$ Numerators by x , you have
	2	$\frac{84}{x-4} = \frac{70}{x-6}$	Then $x - 4 \times 70$
	3	$84 = \frac{70x-280}{x-6}$	Then $84 \times x - 6$,
	4	$84x - 504 = 70x - 280$	Then $\phi 70x$ and -504 ,
	5	$84x - 70x = 504 - 280 = 224$	that is,
	6	$14x = 224$	Therefore,
	7	$x = \frac{224}{14} = 16$	<i>Ans.</i>

Tyr. And can you prove this last Example *synthetically*, as you did in *Multiplication*?

Pbi. Yes, most certainly, if the Work be done right.

A *synthetical Proof* of EXAMPLE 4.

You see that x is $= 16$, then $\frac{84x}{x-4}$, that is, $\frac{84 \times 16}{16-4}$
 $= 112$, the first Side of the *Equation*. Again, $\frac{70x}{x-6}$
 that

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that is, $\frac{70 \times 16}{16 - 6} = 112$. Consequently therefore,

$$\frac{84x}{x-4} = \frac{70x}{x-6} \text{ Q. E. D.}$$

Tyr. Not one Thing that you have demonstrated pleases me better, nor gives me greater Satisfaction.

Phi. I will give you an Example or two more.

EXAMPLE 5.

$$\begin{array}{l|l} \text{Let } 1 & ddx + ddbxx - ddx = ddfx + ddffx. \\ & \text{Then } \div dd \\ 2 & xx + bxx - x = fx + ff x. \text{ Then } \div x \\ 3 & x + bx - 1 = f + ff. \end{array}$$

Here indeed the known Quantities and the unknown Quantity (which we suppose is represented by x) both possess the first Side of the Equation, in Order therefore to let the unknown Quantity x possess one Side by itself, do thus: Let the whole Equation be divided by the known Quantity or Quantities, and then will the unknown Quantity x be equal to the Quotient of such Division. As,

EXAMPLE 6.

Suppose in any Equation it should so fall out, that $xd - xb = c + g$, what is x equal to?

$$\begin{array}{l|l} \text{Let } 1 & xd - xb = c + g. \text{ Then } \div d - b \\ 2 & x = \frac{c + g}{d - b} \text{ Ans.} \end{array}$$

EXAMPLE 7.

Again, Suppose in trying to discover the unknown Quantity, all the Quantities happen to fall together, so as there is no Sign of Equality between, then,

○

then, in Order to form an *Equation*, make such Quantities the first Side, and put a Cypher on the second Side of the *Equation*, so will the unknown Quantity be discovered. Thus,

$$\begin{array}{l|l} \text{Suppose} & \begin{array}{l} 1 \quad 12x - 312, \text{ then} \\ 2 \quad 12x - 312 = 0; \text{ that is,} \\ 3 \quad 12x = 312. \text{ Therefore,} \\ 4 \quad x = \frac{312}{12} = 26. \end{array} \end{array}$$

N. B. See the 9th and 10th Steps of *Problems* 18. *Dialogue* 10.

EXAMPLE 8.

$$\begin{array}{l|l} \text{Let} & \begin{array}{l} 1 \quad 5xxd + 8xx = xbc + 4b, \text{ then } \div x \\ 2 \quad 5xd + 8x = bc + 4b, \text{ then } \div 5d + 8 \\ 3 \quad x = \frac{bc + 4b}{5d + 8} \text{ Ans.} \end{array} \end{array}$$

S E C T. VI.

How to convert or turn EQUATIONS into ANALOGIES, and the contrary.

Tyr. I Imagine that this *Section* depends upon a true Knowledge of the Nature of *Proportion*; does it not?

Pbi. Most certainly, and therefore from what has been laid down in *Dial* 7. *Sect.* 2. it will be easy to convert any *Equation* into an *Analogy*, or right *Proportion*; and especially since I shall take some of the same *Equations*, and refer you back to the former Work, to confirm you the better in what you are doing.

OBSERV.

OBSERV. I.

When any Equation (not having Fractions) is given to be converted into an Analogy, then it will be, as any of the Quantities or Factors on one Side are to any other on the other Side; so will the remaining Quantity or Quantities on the same Side, be to the remaining ones on the other Side, and vice versa.

EXAMPLE I.

Let the Equation be $xd = cb$. See the third Step in the Demonstration. Dial. 7. Sect. 2.

Let	1	$xd = cb$, then
	2	As $x : c :: d : b$. Or
	3	As $x : b :: c : d$. &c. For the 1 Term
	4	$x \times$ (the 4) $d =$ (2) $c \times$ (3) b . Conse-
		quently,
	5	$xd = cb$.

Tyr. I perceive then, this is but a common Proof to Proportion.

Phi. Nothing more; for if you compare this Example with the six Observations laid down in Dial. 7. Sect. 2. you may make a great many more Steps of it than I have done.

Tyr. I see plainly the Manner of turning Equations into Analogies when both the Sides are whole Quantities; but suppose one Side be a whole Number or Quantity, and the other a Fraction?

Phi. Then you are to proceed as follows.

OBSERV. 2.

When any whole Number, or Quantity in an Equation is made equal to a Fraction, whose Numerator

merator has two Quantities and the Denominator but one; then break the Numerator into two such Parts, which multiplied together, will produce the same, and make those Parts the *Means*; then make the whole Quantity, and the Denominator of the *Fraction* the *Extremes*. Or in other Words, make the whole Quantity the first Term in the *Rule of Three*, the Denominator of *Fraction* the fourth, and the Numerator divided into two Parts as before directed, make the second and third Term.

EXAMPLE 2.

Let	1	$x = \frac{bc}{d}$	Then
	2	$x : b :: c : d$	Or
	3	$x : c :: b : d$	That is,
	4	$xd = bc$	See <i>Demonstration</i> , Dial. 7. Sect. 2.

Tyr. I understand it very well, but suppose both be *Fractions*, how then?

Phi. Certainly you forget *Tyrunculus*. Pray turn back to the fourth Step of *Dialogue 7. Sect. 2.* for I shall give you the same *Example*. Or if you remember what I told you in *Abbreviations*, you will find the *Analogy* will hold as follows.

OBSERV. 3.

As one Denominator is to the other, so will one Numerator be to the other; or as one Denominator is to its own Numerator, so is the other Denominator to its Numerator, &c. &c.

Ex.

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EXAMPLE 3.

Let

1	$\frac{b}{x} = \frac{d}{c}$	Then as
2	$x : c :: b : d$	Or as
3	$x : b :: c : d$	$\&c. \&c.$

Tyr. I am obliged to you: Have you Nothing more to add?

Phi. I will give you one *Example* by Way of Exercise.

EXAMPLE 4.

Let

1	$xb + xd = bd$	Then as
2	$x : b :: d : b + d$	For
3	$x \times \overline{b+d} = xb + xd$	the first Side. And
4	$b \times d = bd$	the second Side. Or, as
5	$b : x :: b + d : d$	Or, by adding xd to each Side,
6	$xb + 2xd = bd + xd$	Then, as
7	$x : b + x :: d : b + 2d$	Or, taking xd from each Side,
8	$xb = bd - xd$	Then, as
9	$x : d :: b - x : b$	$\&c. \&c.$ consequently
10	$xb + xd = bd$	

Tyr. Then I perceive by the third and fourth Steps, that if one Side of an *Equation* can be divided into two Parts, so as to become *Extremes* (which being multiplied together, will be equal to the Side before it was divided) the other Side being divided in the same Manner, will be the *Means*; will it not?

Phi. Yes, your Notion is right; and I am glad to find you so perfect in what you have done: Therefore I shall bid you *adieu*, and leave you to consider

upon those *Examples*, which you think yourself least acquainted with; and when Opportunity suits, I shall be glad to see you and your Friend *Novitius*, and then we will put these *Examples* in Practice by some *Algebraic Problems*.

Tyr. Sir, I am obliged to you; and I dare say *Novitius* will be as proud of the *Invitation* — But let me beg of you to stay a little longer.

Phi. Not now, *Tyrunculus*, I think I have made you a long Visit; besides, Night comes on apace, and I choose to go.

Tyr. Sir, if you are determined to go, I heartily wish you a good Night, and humbly thank you for your Company, and I intend to do myself the Pleasure of waiting upon you very shortly.

Phi. When you think proper *Tyrunculus*.





CHAPTER III.

DIALOGUE IX.

SECT. I.

*Between PHILOMATHES and TYRUNCULUS,
concerning the Nature of Algebraic Problems, and
how to prepare them for a Solution.*

Tyrunculus returning the Visit to Philomathes.

Tyr.  *Philomathes, your humble Servant,
how do you do?*

Phi. Thank you, *Tyrunculus*, I
am pretty well, and am glad to
see you so.

Tyr. You remember I said it
should not be long before I would call again to see
you; but, perhaps, I am not come at a suitable Time.

Phi. You could not have hit upon it better, *Ty-
runculus*, it suits me quite well, and I was but just
before thinking of, and wishing for you. — Come,
pray sit down, — But where is your Friend *Novitius*,
I expected you both together?

Tyr.

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Tyr. We are obliged to you, Sir, and I asked him to come, as you desired, and he promised to follow me.

Phi. Well, *Tyrunculus*, in the mean Time let me know how you go on, and what Improvement you have made since I saw you last.

Tyr. I am afraid it will not bear too close an Examination: However, that very Night you left me, I looked over the Chief of what you have shewn me, and find myself much more perfect in it.

Phi. You have done well, it is all I required, and you will be the better able to understand the following Problems.

Tyr. I must confess I do not care how soon I begin to try a few Questions, or at least see them wrought, for you must know I am in a Hurry.

Phi. You shall presently; but pray be not so over hasty; *fair* and *softly*, you know, go the furthest; and I have Something to premise first of all, that will be of Service, and help you forward in the Work.

Tyr. Pray what is that?

Phi. It would be requisite that you should be acquainted with the following Observations.

OBSERV. I.

When any Question is given to be answered in an Algebraic Manner, *first*, For the Answer or Number sought, put x : Then proceed according to the Tenor of the Question, to *add*, *subtract*, *multiply*, or *divide*, until you have formed an Equation, which if it has *Fractions* must be cleared according to the Rules laid down in *Multiplication*. *Seet. 4. Dial 8*: This done, proceed to *transpose* according to the Order of *Addition* and *Subtraction* of Equations, and you will (by keeping x on the first Side of the Equation)

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tion) have it equal at last to some known Quantity or Quantities, by which also x will be of Course known, and its Value discovered.

OBSERV. 2.

Though it be customary to use x for the unknown Quantity, yet you may make Use of any other Letter at Pleasure. Some *Analists* use Vowels to represent *unknown*, and Consonants *known* Quantities; but others use them as their own Fancy and Inclination direct: But still you are to observe, the Letter (o) is never used to express a Quantity, (though indeed the Answer would be the same with this as with any other Letter,) and there seems to be a Reason for it, since it is but a Cypher at best without Integers; and therefore, since Nothing cannot be Something, by Reason of its Want or Deficiency, it would be absurd to put it to represent any Number or Quantity; though, as I observed before, it is sometimes used to form an Equation, See Problem 18. Step. 10.

OBSERV. 3.

If to the Sum of any two Numbers you add their Difference, and divide the Whole by 2, the Quotient will be the greater Number. Or if you add the Numbers and their Difference together, and divide by 2, you have the greater Number.

OBSERV. 4.

If from the greater Number you take the Difference of the said Numbers, the Remainder will be the less Number. Or, if you add any two Numbers and their Difference together, and divide the
Sum

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Sum by 2, and then subtract the Difference of the said Numbers from the Quotient, the Remainder will be the less Number.

OBSERV. 5.

When any *Fraction* is given to be divided into two, three, or more Parts, then divide the Numerator, if you can, into such Parts, and let the Denominator remain as it was; and in Case you cannot divide the Numerator into the Parts required, multiply the Denominator into such Parts as are requir'd, and let the Numerator remain as it was; so is the *Fraction* truly divided into such Parts as really as if it had been performed by *Division*, which is sometimes very difficult. Thus, suppose I was to divide $\frac{6}{x}$ into three Parts, I divide 6 by 3, and it is 2; so is $\frac{2}{x}$ the $\frac{1}{3}$ of $\frac{6}{x}$. But suppose it were $\frac{x}{6}$ to be divided into 3 Parts, as I cannot well divide x by 3, therefore I multiply the Denominator 6 by 3, and it is 18; so is $\frac{x}{18}$ the $\frac{1}{3}$ of $\frac{x}{6}$. This I have demonstrated, because you shall seldom meet with it in any Authors, although it is of infinite Service in *Algebra*. I beg therefore you would remember it in particular.

OBSERV. 6.

When any two Numbers are given, and you would express them *literally*, (we will suppose you put x for the greater, and e for the less Number) then will the following Steps be of Service, because they will help you to understand the Nature of a Question, and the sooner to do the same, as being a proper

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er and necessary Exercise of the foregoing Rules,
by teaching you how to express them in their true
Order. Thus,

- 1 Suppose the greater be x ,
- 2 And the less Number e ,
- 3 Then will their Sum be $x + e$,
- 4 Their Difference $x - e$,
- 5 Their Product $x \times e$, viz. xe ,
- 6 Their Quotient of the greater $\div$ by the less is $\frac{x}{e}$,
- 7 The Quotient of the less by the greater is $\frac{e}{x}$,
- 8 The Order of Proportion, as $x : e :: e : \frac{ee}{x}$.
- 9 Or, by putting the less first, as $e : x :: x : \frac{xx}{e}$.
- 10 The Square of the greater xx ,
- 11 The Square of the less ee ,
- 12 The Sum of their Squares $xx + ee$
- 13 The Difference of their Squares $xx - ee$,
- 14 The Sum of their Sum and Difference $2x$,
- 15 The Difference of their Sum and Difference $2e$
- 16 The Product of their Sum and Difference $xx - ee$,
- 17 The Square of their Sum $xx + 2xe + ee$,
- 18 The Square of their Difference $xx - 2xe + ee$,
- 19 The Square of their Product $xxee$,
- 20 The Cube of the greater xxx , or x^3 ,
- 21 The Cube of the less eee , or e^3 , &c.

These being understood, you may proceed to the
working of the following *Algebraic Problems*.

DIA-

DIALOGUE X.

SECT. I.

ALGEBRAIC PROBLEMS, *or the Solution of Questions producing SIMPLE EQUATIONS.*

Between PHILOMATHES, TYRUNCULUS, and NOVITIUS; being a proper Exercise of all the foregoing Rules.

Tyr. THIS is that Part of *Algebra* that I have so long wish'd to be trying at, and to which by your kind Assistance, *Philomathes*, I am at last happily arrived to.

Phi. I am as much satisfied, and take as great Pleasure in your Progress as you possibly can, and I doubt not of your Understanding the Manner of working the *Problems* in a short Time. Only take Care to mind the Steps in the *numerical* Work, and you will soon understand the *literal*, for I shall endeavour to make the Steps alike if I can. And though you be perfect in the Chief of what you have done, yet give me Leave once more to remind you of these three Things, *viz.* That this Character (ϕ) in any Step shews you that the Number or Quantity before which it is placed is transposed in the next Step to the other Side of the *Equation*; this (Q .) signifies by the *Question*; and lastly, to remember that to take the $\frac{1}{2}$ or $\frac{1}{3}$, &c. of any *Fraction* is only to multi-
ply

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ply the Denominator by 2, 3, &c. which is the same as to divide the Numerator by the same Figures. These being observed, we will proceed to

PROBLEM I.

What Number is that which being multiplied by 12, and having 18 added to the Product, the Sum will be 294?

Numerical Solution.

Put	1	x for the Number, this $\times 12$
	2	$12x$, add 18, it is
	3	$12x + 18$. This $Q. = 294$. Then $\phi 18$
	4	$12x = 294 - 18$; that is,
	5	$12x = 276$. Then is
	6	$x = \frac{276}{12} = 23$ Ans.

Literal Solution.

Let $b = 12$, $c = 18$, $d = 294$.

1	x as before $\times b$
2	xb add $+ c$
3	$xb + c$. This $Q. = d$
4	$xb + c = d$. Then ϕc
5	$xb = d - c$
6	$x = \frac{d - c}{b} = 23$ Ans.

PROBLEM II.

What Number is that to which if I add 24, then from that Sum subtract 8, and multiply the Remainder by 5, the Product will be 320?

P

Numerical

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Numerical Solution.

Put	1	x for the Number, then
	2	$x + 24$, then $- 8$,
	3	$x + 24 - 8$; this $\times 5$, is
	4	$5x + 120 - 40$. Then Q .
	5	$5x + 120 - 40 = 320$. By $\phi 40$
	6	$5x + 120 = 320 + 40$. Then $\phi 120$
	7	$5x = 360 - 120 = 240$. Therefore
	8	$x = \frac{240}{5} = 48$ Ansf.

Literal Solution.

	Let	$b = 24, c = 8, d = 5, f = 320$.
Put	1	x as before
	2	$x + b$
	3	$x + b - c$. This $\times d$
	4	$dx + db - dc$ Whence Q .
	5	$dx + db - dc = f$
	6	$dx + bd = f + dc$
	7	$dx = f + dc - bd$
	8	$x = \frac{f + dc - bd}{d} = 48$

But it is to be noted that all such like Questions as *this* may be performed both shorter and easier, by working only with the Difference of the Numbers, and not the Numbers themselves. Thus, you are desired in the *Problem* to add 24, then subtract 8. Now it is evident that $24 - 8 = 16$; therefore if you work only with 16, by adding it to the unknown Quantity, it must be the same as to add 24 and subtract 8. So that the *Problem* may be read thus :

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PROBLEM II. in other Words.

What Number is that to which if I add 16, and multiply that Sum by 5, the Product will be 320?

Numerical Solution.

Put	1	x as before, then $+ 16$,
	2	$x + 16$. This $\times 5$,
	3	$5x + 80$. Whence Q .
	4	$5x + 80 = 320$. Then $\phi 80$
	5	$5x = 320 - 80 = 240$. Then
	6	$x = \frac{240}{5} = 48$, as before.

Literal Solution.

Let $b = 16$, $f = 320$, $= d 5$

	1	x
	2	$x + b$. This $\times d$
	3	$xd + bd$ Q .
	4	$xd + bd = f$
	5	$xd = f - bd$
	6	$x = \frac{f - bd}{d} = 48$, as before

Tyr. This is much shorter and better indeed as you observe, and I begin to understand Something of the *literal* Operation, as well as the *numerical*; but I must needs say, I like the *numerical* best, I think it is the plainest for Learners.

Phi. Most are apt to say so indeed; but when once the other Way is known, you will like that as well, and to be sure it is the shorter of the two, but I will not say the easier. However, I will perform all the *Problems* numerically, and some of them literally; and pray let me advise you to read over every

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every Question, at least twice or thrice, that you may understand the Nature of it the better, for when once you have a true and just *Idea*, of the Intent and Meaning of it, you may be said to have half done it.

PROBLEM III.

Says Alexander to Ephestion, I am older than you by two Years. Clitus hearing it, said, I know I am older than both of you by four Years. The Philosopher, Callisthenes, being present, said, I remember I have heard my Father, who is now ninety-six Years, say, that he is as old as you all; I demand then the Age of Alexander, Clitus, and Ephestion.

Numerical Solution.

Put	1	x Ephestion, then will
	2	$x + 2$ be Alexander's. Then $+ 4$
	3	$2x + 6$ Clitus. These added
	4	$4x + 8$ their Sum; whence Q.
	5	$4x + 8 = 96$. Then $\phi 8$
	6	$4x = 96 - 8 = 88$. Therefore
	7	$x = \frac{88}{4} = 22$ Ephestion's Age
	8	x viz. $22 + 2 = 24$ Alexander's
	9	$xx + 6 = 50$ Clitus's

Literal Solution.

Let $b = 2$, $d = 4$, $f = 96$

1	x Ephestion's,
2	$x + b$ Alexander's,
3	$2x + b + d$ Clitus's,
4	$4x + 2b + d$ Sum. Then Q.
5	$4x + 2b + d = f$. Then $\phi 2b + d$
6	$4x = f - 2b - d$

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$$\begin{array}{l|l} 7 & x = \frac{f - 2b - d}{4} = 22 \text{ Ephestion's} \\ 8 & x + b = 24 \text{ Alexander's} \\ 9 & 2x + b + d = 50 \text{ Clitus's.} \end{array}$$

Tyr. Mighty pretty; but why do you begin with *Ephestion* rather than *Alexander*?

Phi. It would have been the same had I began with *Alexander's* Age; only then the 2d Step would have been $x - 2$ for *Ephestion*, and the 3d Step $2x - 2 + 4$; and so it would have occasioned more Work, but now they are all affirmative.

Tyr. I am satisfied, and begin to see a little more into it.

Phi. There is no Fear of your Understanding it, if you mind.

PROBLEM IV.

Three Persons, A, B, and C, trade and gain 3000 £. the Share of A is to be but Half the Share of B, and the Share of B one Third the Share of C; I demand each Man's Share?

Now to avoid Fractions, I begins with A first; for if I put x for B, then A must be $\frac{x}{2}$ and C $3x$. Therefore

Numerical Solution.

Put	1	x for A, then is
	2	$2x$ B's Share, and
	3	$6x$ C's Share. These added make
	4	$9x$ their Sum. Whence Q,
	5	$9x = £. 3000.$ Therefore

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6	$x = \text{£. } \frac{3000}{9} = \text{£. } 333 \text{ } 6 \text{ s. } 8 \text{ d. } A,$
7	And by the 2d Step 666 13 s. 4 d. B,
8	And by the third 2000 C,
9	Their Sum = 3000.

Literal Solution.

	As x is put for A , and the other is double and treble, the first four Steps will be the same as in the <i>numerical</i> . Now let
	$b = \text{£. } 3000$
5	$9x = b$
6	$x = \frac{b}{9} = \text{£. } 333 \frac{1}{3}$
7	$2x = 666 \frac{2}{3}$
8	$6x = 2000$
9	Sum = 3000 £. as before.

Do yo understand these Operations ?

Tyr. The *literal* Part I am not at present so much acquainted with, but the *numerical* appears quite plain and easy to me : I heartily wish *Novitius* was here, he would be so pleas'd to see some of those Questions which puzzle him, demonstrated in so easy a Manner.

Phi. — There is a young Gentleman now coming up the Walk.

Tyr. Perhaps it is he——It is so—I will go to the Door for I know he is quite bashful——

Phi. Stay——give me Leave, *Tyrunculus* ; it will look better in me, and he will take it kinder at my Hands.——

Nov. Your humble Servant, *Philomathes* : Pray is *Tyrunculus* here ?

Phi. He is, pray come in, *Novitius*.

Nov.

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Nov. Sir——Friend *Tyrunculus*, how fare you?

Tyr. I am a little vexed with you for staying.

Nov. I ask *Philomathes's* Pardon in particular—I was unexpectedly prevented by an Acquaintance.

Phi. Well, *Novitius*, we will not use superfluous Ceremonies at this Time: Pray sit down, I am glad to see you.

Nov. Sir I thank you.

Phi. Your Friend *Tyrunculus* was saying you had some Questions to ask me that had puzzled you pretty much, pray, what are they?

Nov. Only some few of *Mr. Cocker's* and *De-Billy's*, for several of them are so contracted in the Work that I cannot understand them.

Phi. To do Justice to the first Author, I know not a prettier Piece for Learners on the first four Rules of *Algebra*; but I confess he is a little dark in some of the Operations. Come, I have him by me, and pray do you look out those Questions that puzzle you most, and we will work them more plainly to your Understanding.

Nov. Please then to begin with his fifth Question.

Phi. There are *Fractions* concerned in that; therefore I think we had better begin with the more easy ones first, and take the harder as they come in Course.

Nov. It is true; do so if you please.

Phi. Observe then.

• P R O •

Nov.

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* PROBLEM V. — Cocker's 8th Question.

A Labourer received 2 £ . 8s. for threshing 60 Quarters of Corn, viz. Wheat and Barley; for the Wheat he received 12 Pence a Quarter, and for the Barley 6 Pence: How many Quarters did he thresh of each.

Numerical Solution.

- 1 For the Quarters of Wheat put x ,
- 2 Then as both together are but 60, the Barley must be $60 - x$.
- 3 Now x Quarters of Wheat, at 12d. a Quarter, is $12x$.
- 4 And $60 - x$ Quarters of Barley, at 6d. is $360 - 6x$.
- 5 These two are equal Q. to 48s. or 576 Pence; whence $12x + 360 - 6x = 576$.
- 6 Then ϕ 360, $12x - 6x = 576 - 360$;
- 7 That is, $6x = 576 - 360 = 216$.
- 8 Therefore $x = \frac{216}{6} = 36$ Quarters of Wheat.
- 9 And by 2d Step $60 - x$, or $60 - 36 = 24$ the Barley.

Literal Solution.

Let $b = 576$ Pence, $c = 60$, $d = 12$, $f = 6$ Pence.

- 1 x Wheat.
- 2 $c - x$ Barley. Then $x \times d$ is
- 3 dx for the Wheat, and $f \times c - x$ is

[4]

*Those Problems that have an Asterisk before them, are inserted by the Desire of several young *Algebraists*, who wanted a plainer and easier Demonstration than in the Original: And to such as are unacquainted with either of the Authors, they will be equally serviceable to, as if they were new ones.

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- 4 | $fc - fx$ for the Barley. Whence Q.
 5 | $dx + fc - fx = b$, or 576 Pence. Then
 6 | $dx - fx = b - fc$. Therefore
 7 | $x = \frac{b - fc}{d - f}$; that is, $x = \frac{576 - 360}{12 - 6} = 36$ as
 above Wheat,
 8 | And $c - x$, or $60 - 36 = 24$, the Barley.

PROOF.

36 Quarters, at 1s. = 36s.

24 Quarters, at 6d. = 12s.

48s.

Nov. I understand it quite well.

Tyr. So do I both the Ways.

Phi. I shall not write against the *literal* any more, but leave you to compare it with the *numerical* Work.

* PROBLEM VI. — Cocker's 10th Question.

A Gentleman hired a Servant for 40 Days upon this Condition, that every Day he wrought he was to receive 20 Pence, and for every Day he was idle, and did no Work, he was to pay 8 Pence. Now at the End of the Time he received 15s. 4d. How many Days did he work, and how many was he idle?

Numerical Solution.

- 1 | For the Days he wrought put x
 2 | Then will what he play'd be $40 - x$.
 3 | Now x Days wrought, at 20d. a Day, is $20x$,
 4 | And $40 - x$ play'd, at 8d. a Day, is $320 - x$.
 5 | Subtract the 4th Step from the 3d Step, it is
 $20x - 320 + 8x$.

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- 6 This being his Due, is (Q.) $= 15\text{ s. } 4\text{ d.}$
- 7 Whence $20x - 320 + 8x = 184\text{ d.}$
- 8 That is, $28x - 320 = 184,$
- 9 That is, $28x = 184 + 320 = 504.$
- 10 Therefore $x = \frac{504}{28} = 18\text{ Days Ans.}$
- 11 And by 2d Step, $40 - x = 22\text{ idle.}$

Literal Solution.

Let $b = 40, c = 20, d = 8,$ and $f = 184\text{ Pence,}$
viz. $15\text{ s. } 4\text{ d.}$

- 1 x Days wrought,
- 2 $b - x$
- 3 cx
- 4 $db - dx.$ Whence
- 5 $cx - db + dx = f$
- 6 $cx + dx = f + db$
- 7 $x = \frac{f + db}{c + d} = 18,$ as before;
- 8 $b - x = 22\text{ idle.}$

PROOF.

	£.	s.	d.
18 Days, at 20 d. a Day,	1	10	0
22 idle, at 8 d. a Day,	0	14	8
Due at last	0	15	4

* PROBLEM VII. — Cocker's 6th Question.

Two Persons, A and B, thus discoursed of their Money:
 Says A to B, give me 3 of your Crowns, and I shall
 have as many as you; and says B to A, give me 3 of
 yours, and I shall have 5 Times as many as you. I
 demand the Number each had?

Numerical

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Numerical Solution.

- 1 For the N^o. *A* had at first put x ,
- 2 Then by *B*'s giving him 3, he will have $x+3$;
- 3 And as this makes *B*'s equal, *B* will also have $x+3$,
- 4 And therefore consequent *B* had at first $x+6$,
- 5 And if *A* had given him 3, he would then have $x+9$,
- 6 And by the same Reason *A* would have but $x-3$
- 7 Now *B* (Q.) should have 5 Times this Number, viz. $5x-15$
- 8 Whence this Equation, $5x-15 = x+9$.
- 9 Then ϕ x and -15 , it will be $5x-x = 9+15$,
- 10 That is $4x = 24$.
- 11 Therefore $x = \frac{24}{4} = 6$ *A*'s.
- 12 And by the 4th Step $x+6 = 12$ *B*'s.

Literal Solution.

Let $e = 3$ Crowns; then by comparing the Steps,

- 1 x for *A*,
- 2 $x + e$,
- 3 $x + e$,
- 4 $x + 2e$,
- 5 $x + 3e$,
- 6 $x - e$,
- 7 $5x - 5e$,
- 8 $5x - 5e = x + 3e$,
- 9 $5x - x = 3e + 5e$,
- 10 $4x = 8e$

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$$\left| \begin{array}{l} \text{II} \\ \text{12} \end{array} \right| \begin{array}{l} x = \frac{8e}{4} = 6, A's. \\ x + 2e = 12, B's. \end{array}$$

PROOF.

$$\begin{array}{l} A \ 6 + 3 = 9, \\ B \ 12 - 3 = 9, \text{ but} \\ A \ 6 - 3 = 3, \text{ and} \\ B \ 12 + 3 = 15 = 3 \times 5. \end{array}$$

Nov. I am perfectly satisfied, it is done so plain, and the Steps are built upon Reason itself.

Phi. It is upon Reason itself that *Algebra* depends; and a Question laid down in a good and clear Light, is the Learners chief Guide. But now for some that require *Fractions*, for I suppose you understand them!

Nov. Yes, I think I do pretty well.

Phi. Here follows then.

* PROBLEM VIII. — Cocker's 5th.

There is a Fish whose Head is 9 Inches long, and his Tail is as long as his Head, and half as long as his Body, and his Body is as long as his Tail and his Head; I demand the whole Length of the Fish.

$$\left| \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right| \begin{array}{l} \text{For the Length of his Body put } x \\ \text{Then will his Tail be } \frac{x}{2} + 9, \\ \text{And his Body should be as long as his Tail and Head, viz. } \frac{x}{2} + 9 + 9. \\ \text{Whence (Q.) this Equation } x = \frac{x}{2} + 18, \end{array}$$

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- 5 This $\times$ Denominator 2, is $2x = x + 36$.
- 6 Then ϕ x , $2x - x = 36$,
- 7 That is, $x = 36$ his Body,
- 8 And by 2d. Step, $\frac{x}{2} + 9 = 27$ his Tail;
- 9 To which add 9 his Head,
- 10 Their Sum is 72 his Length.

Literal Solution.

Let $b = 9$,

- 1 x ,
- 2 $\frac{x}{2} + 2b$,
- 3 $\frac{x}{2} + 2b$,
- 4 $x = \frac{x}{2} + 2b$,
- 5 $2x = x + 4b$,
- 6 $2x - x = 4b$,
- 7 $x = 4b = 36$, Body.
- 8 $\frac{x}{2} + b = 27$, Tail.

PROBLEM IX. His 12th Question.

One asked a Shepherd the Price of his 100 Sheep. No, replied he, if I had as many more, and half as many more, and seven Sheep and an half, I should then have 100. I demand the Number he had.

Numerical Solution.

- Put
- 1 x for the Number, then
 - 2 x as many more, and
 - 3 $\frac{x}{2}$ is half as many more, and

Q

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4	$7\frac{1}{2}$	These four Steps added
5	$2x + \frac{x}{2} + 7\frac{1}{2}$	Whence (Q.)
6	$2x + \frac{x}{2} + 7\frac{1}{2} = 100$	This reduced
7	$4x + x + 15 = 200$	Then ϕ 15,
8	$5x = 200 - 15 = 185$	Therefore
9	$x = \frac{185}{5} = 37$	Answer.

Literal Solution.

Let $b = 7\frac{1}{2}$, $c = 100$.

1	x ,
2	x ,
3	$\frac{x}{2}$,
4	b ,
5	$2x + \frac{x}{2} + b$ Sum (Q.)
6	$2x + \frac{x}{2} + b = c$,
7	$4x + x + 2b = 2c$,
8	$5x = 2c - 2b$,
9	$x = \frac{2c - 2b}{5} = 37$. Ans.

* PROBLEM X. His 9th Question.

*A Gentleman bought a Cloak of a Salesman, which cost him 3*l*. 10*s*. and after he had bought it he desired the Salesman to tell him ingenuously what he gain'd by it, who answered I gain just $\frac{1}{4}$ Part of what it cost me. It is demanded what the Cloak cost the Salesman.*

Numerical Solution.

1	For what the Cloak cost the Salesman put x ,
2	Then his Gain or Profit will be $\frac{x}{4}$

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3 These added are equal to what he sold it for,

$$x + \frac{x}{4},$$

4 Therefore (Q.) $x + \frac{x}{4} = 70s.$

5 This reduced, $4x + x = 280;$

6 That is, $5x = 280.$

7 Therefore $x = \frac{280}{5} = 56s.$ Ans.

Literal Solution.

Let $s = 70.$

1 $x,$

2 $\frac{x}{4},$

3 $x + \frac{x}{4}$

4 $x + \frac{x}{4} = s,$

5 $4x + x = 4s,$

6 $5x = 4s,$

7 $x + \frac{4s}{5} = 56$ as before.

PROOF.

Cost him $56s.$

Gained $\frac{1}{4} = 14s.$

Sum $= 70s.$ sold it for.

*** PROBLEM XI. — His 11th Question.**

A Person in the Afternoon being ask'd what a Clock it was, answered, that $\frac{3}{4}$ of the Time past from Noon, was equal to $\frac{3}{8}$ of the Time to Midnight. Now
allowing

Q 2

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allowing 12 Hours to the Day, and beginning to reckon from Noon, I demand what Hour it was when the Question was asked.

Numerical Solution.

- 1 For the Hours sought from Noon put x ,
- 2 Then will the Time to Midnight be $12 - x$,
- 3 Now $\frac{2}{3}$ of x is $\frac{3x}{5}$,
- 4 And $\frac{2}{3}$ of $12 - x$ is $\frac{60 - 5x}{8}$.
- 5 These (Q.) are equal. Whence $\frac{60 - 5x}{8}$.
- 6 This reduced, viz. first $\times$ Denominator 5, is
 $3x = \frac{300 - 25x}{8}$.
- 7 Then this $\times$ the Denominator 8 will be $24x = 300 - 25x$,
- 8 Then $49 - 25x$ it is $24x + 25x = 300$,
- 9 That is, $49x = 300$.
- 10 Therefore $x = \frac{300}{49} = 6 \frac{6}{49}$.
- 11 And by 2d Step $12 - x = 5 \frac{43}{49}$.
 So that it was $\frac{6}{49}$ past 6, that is $7' 20'' \frac{40}{49}$ past 6.

PROOF.

H. / "

$$\begin{array}{r} 6 \frac{6}{49} = 6 \ 7 \ 20 \ \frac{40}{49} \\ 5 \frac{43}{49} = 5 \ 52 \ 39 \ \frac{19}{49} \end{array}$$

Sum = 12 Hours.

Literal

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Literal Solution.

Let $b = \frac{3}{5}$, $c = \frac{5}{8}$, and $d = 12$

1	x as above,
2	$d - x$,
3	bx ,
4	$cd - cx$,
5	$bx = cd - cx$,
6	$bx + cx = cd$,
7	$x = \frac{cd}{b+c}$, viz. $x = \frac{5}{8} \times 12 \div \frac{3}{5} + \frac{5}{8} = 6$ $\frac{48}{392}$ or $6 \frac{6}{49}$ as before.

Nov. This is plain upon my Word, and I perceive $6 \frac{6}{49}$ is the same Answer as Mr. Cocker's; but still I never could know, nor do I yet rightly apprehend from whence this $\frac{48}{392}$ proceeds; and therefore I never could make the Answer chime in with his.

Phi. I must own that a Man had need to understand *Fractions* quite perfectly to find such Things out to his own Satisfaction; for it is not every *Learner* can do it. Observe then, the Answer is $x = \frac{3}{8} \times 12 \div \frac{3}{5} + \frac{5}{8}$. Now $\frac{3}{8} \times 12 = \frac{36}{8} = \frac{9}{2}$. And $\frac{9}{2} + \frac{5}{8}$ being reduced to a common Denominator and added, their Sum will be $\frac{45}{8}$ and $\frac{25}{8}$ (that is $\frac{70}{8}$) by which divide $\frac{60}{8}$ it is $\frac{2400}{70} = 6 \frac{6}{49}$ as before; and therefore the Hour to Midnight is $5 \frac{36}{49} = 5 \frac{6}{7}$.

Tyr. There, *Novitius*, is that plainly demonstrated or not?

Nov. Quite plain indeed! And now *Philemathis* I will ask you to work the 6th Question and no more, for the 5th Step, I never could apprehend it, it has puzzled me many a Time.

Q 3

Phi.

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Phi. I must confess I was a long Time in finding it out myself, and tho' the same Question be in Mr. *Ward*, it did not answer my Expectation; for I found it out by Mr. *Cocker's* Method at last.

Nov. But then he abbreviates his Numbers and Fractions, and gives no Reason, and this puts the Learner to a Stop. *

Phi. It is true he does so. Well, *Novitius*, all I can say, is, I will work the same Question numerically, and leave you to judge which of the three Ways are the easiest to be understood, supposing you had them now all before you.

* PROBLEM XII. *Cocker's* 6th, and *Ward's* 31st Question.

A Father lying at the Point of Death, left his three Son, A, B, and C, his Estate as follows. To A he gave $\frac{1}{2}$ wanting 44 £. to B he gave $\frac{1}{3}$ and 14 £. over; and to C he gave the Remainder which was 82 £. less than the Share of B. I demand the Father's Estate in ready Money?

Numerical Solution.

- 1 For the Estate put x ,
- 2 Then will *A's* Legacy be $\frac{x}{2} - 44$,
- 3 And *B's* will be $\frac{x}{3} + 14$,
- 4 And *C's* being 82 less, is $\frac{x}{3} + 14 - 82$.
- 5 These Fractions being first reduced to a common Denominator, and added, the Sum of the Whole is $\frac{21x}{18}$, or $\frac{7x}{6} + 28 - 126$. This (Q.) equal to the Estate or x .

[6]

* See the Preface.

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- 6 | Whence this Equation $\frac{7x}{6} + 28 - 126 = x$,
 Now $28 - 126$ being the same as -98 , it will
 7 | be $\frac{7x}{6} - 98 = x$.
 8 | This reduced $7x - 588 = 6x$.
 9 | Then $\phi 6x$, and 588 , it will be $7x - 6x = 588$,
 10 | That is, $x = 588$ Estate.

PROOF.

A's Share $\frac{1}{2} - 44 = 250 = \frac{x}{2} - 44$ by 2d Step.

B's $\frac{1}{3} + 14 = 210 = \frac{x}{3} + 14$ by 3d. Step.

C's $82\text{£. less than B,} = 128 = \frac{x}{3} + 14 - 82$ by 4th
 Step.

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Nov. I heartily thank you, kind *Philomathes*;
 then I perceive that Mr. Cocker's 5th Step before
 Abbreviation, was $\frac{21a}{18} + 2c - b - d$.

Phi. You are right, and one would think a
 Learner might easily perceive it if he would be di-
 ligent.

Nov. You know, Sir, a small Matter turns the
 Learner quite out of his direct Path. I have now with
 me *James de Billy's Algebra*, but it is wrought in so
 odd a Manner, that I cannot make out any Thing
 by it. I could wish you would work some of his
 Questions numerically.

Phi. Which would you have me begin with?

Nov. His 2d Question if you please.

Phi. I will work several of them.

* PRO-

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* PROBLEM XIII. — *J. de Billy's 2d Question.*

A Hare is 100 Yards distant from a Dog, and both starting together, the Dog ran $2\frac{1}{2}$ Times faster than the Hare: It is demanded how far the Hare will have run before the Dog overtakes her?

Numerical Solution.

- 1 For the Yards the Hare ran put x ,
- 2 Then will the Dog when he overtakes her have run $100 + x$.
- 3 Now because he runs $2\frac{1}{2}$ Times faster than the Hare, take any two Numbers bearing the like Proportion as 5 and 2, then.
- 4 By the Rule of 3, as $5 : 2 :: 100 + x : x$,
- 5 Multiplying *Means* and *Extremes*, $5x = 200 + 2x$.
- 6 Then ϕ $2x$, — — — $5x - 2x = 200$,
- 7 That is, — — — $3x = 200$.
- 8 Therefore $x = \frac{200}{3} = 66\frac{2}{3}$
- 9 And by 2d Step, $100 + x = 166\frac{2}{3}$.
So that the Hare run $66\frac{2}{3}$ Yards, and the Dog $166\frac{2}{3}$.

* PROBLEM XIV. — His 11th Question.

A certain Man agreed with his Servant for 12 Months Service to give him 10 Crowns and a new Coat; but disagreeing, he at the End of 7 Months gives him the Coat and two Crowns: I demand the Value of the Coat?

Numerical

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Numerical Solution.

- 1 For the Value of the Coat put x ,
- 2 Then will this and 10 Crowns be his Year's Wages, $x + 10$.
- 3 Now to find one Month's Wages say,
As $12 : x + 10 :: 1 : \frac{x + 10}{12}$,
- 4 And because he had 2 Crowns and the Coat for 7 Months, say,
As $7 : x + 2 :: 1 : \frac{x + 2}{7}$.
- 5 These being both 1 Month's Wages are equal.
Whence $\frac{x + 2}{7} = \frac{x + 10}{12}$.
- 6 This $\times$ the Denominator 7, is $x + 2$
 $= \frac{7x + 70}{12}$.
- 7 This $\times$ 12, the other Denominator, $12x + 24 = 7x + 70$, then ϕ $7x$ and 24,
- 8 $12x - 7x = 70 - 24$, that is,
- 9 $5x = 46$,
- 10 Therefore $x = \frac{46}{5} = 9 \frac{1}{5}$ Crowns.
So that the Value of the Coat was $9 \frac{1}{5}$ Crowns, or 46 Shillings, to which add two Crowns, is 56 Shillings that he had for his 7 Month's Service; and for the Year's Service, had he staid, $x + 10$ Crowns by 2d Step, viz. 96 Shillings.

* PROBLEM XV. — His 16th Question.

A Man having a certain Number of Crowns about him, desired a Stander-by to guess at them, who said, you have 600 perhaps. No, says he; but if to what I have were added $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, and from what I have

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*have were subtracted $\frac{1}{12}$, I should then have 600:
I demand the Number he had about him?*

Note, As in *Problem 2*, so here also make the Subtraction first, and work with the Difference only. Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, reduced to a common Denominator, will be $\frac{3}{4}$, or $1 \frac{2}{4} = 1 \frac{1}{2}$; from which take $\frac{1}{12}$, there remains 1 whole Integer. So that supposing I were to put x for the Number, then $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ added, and $\frac{1}{12}$ taken from it, x will be the Difference; therefore $2x = 600$, and $x = 300$ the Number. But see the Whole as follows.

- | | |
|---|--|
| 1 | For the Number sought put x , |
| 2 | Then $\frac{x}{2}, \frac{x}{3}, \frac{x}{4} = \frac{25x}{24}$, that is, $= x + \frac{x}{12}$ as before. |
| 3 | Add these, it is $2x + \frac{x}{12}$, |
| 4 | From which take $\frac{1}{12}$, viz. $\frac{x}{12}$, |
| 5 | There remains only $2x$. Whence (Q.) $2x = 600$. |
| 6 | And therefore $x = 300$ <i>Ans.</i> |

Now. I heartily thank you, this appears plain enough to me. I will trouble you to work one more if you please, and that is his 7th Question, for this is quite dark to me.

* PROBLEM XVI. — His 7th Question.

A certain Person bought a Number of Ells of Velvet, which he sold again; he bought 5 Ells for 7 Crowns, and sold seven Ells for 11 Crowns, and gained 100 Crowns in so doing: I demand how many Ells there were in all?

Numerical

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Numerical Solution.

- 1 For the Number put x ,
 - 2 Then, if 5 Ells be 7 Crowns, what will x be? $\frac{7x}{5}$,
 - 3 Then, if 7 Ells be sold for 11 Crowns, what will x fetch? $\frac{11x}{7}$,
 - 4 The Difference of these shew his Gain $\frac{6x}{35}$.
 - 5 This (Q.) = 100 Crowns. Whence $\frac{6x}{35} = 100$
 - 6 This reduced $6x = 3500$.
 - 7 Therefore $x = \frac{3500}{6} = 583 \frac{1}{3}$.
- So that the Number was $583 \frac{1}{3}$ Ells bought and sold.

PROOF.

If 5 Ells be 7 Crowns, what is $583 \frac{1}{3}$? *Ans.* $816 \frac{1}{3}$ Cr.
 If 7 Ells be 11 Crowns, what is $583 \frac{1}{3}$? *Ans.* $916 \frac{1}{3}$ Cr.

He gained 100 Cr.

PROBLEM XVII.

There are two Numbers whose Sum is 240, and the Greater has the same Proportion to the Less as 7 to 3: I demand the Numbers?

Numerical Solution.

- 1 For the Greater put x ,
- 2 Then will the Less be $240 - x$.

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* Note, The Fractions $\frac{7x}{5}$ and $\frac{11x}{7}$ are first reduc'd to a common Denominator, and then the Difference you will find is $\frac{6x}{35}$.

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- 3 These (Q.) have this Proportion,
as $7 : 3 :: x :: 240 - x$.
- 4 Then multiplying *Means* and *Extremes*,
 $3x = 1680 - 7x$.
- 5 Then $\phi - 7x$, it is $3x + 7x = 1680$.
- 6 That is, $10x = 1680$.
- 7 Therefore $x = \frac{1680}{10} = 168$ G. Number.
- 8 And by 2d Step $240 - x = 72$ L. Number.

* PROBLEM XVIII.

A certain Toper went to an Alehouse, and borrow'd as much Money as he had about him, out of which he spent a Shilling; then he went to a 2d Alehouse, and borrowed as much as he had then about him, and spent a Shilling; and in like Manner he went to a 3d and 4th Alehouse, borrowing as much as he had left at the former, and spent a Shilling; but after he had spent a Shilling at the 4th Alehouse he had Nothing left: It is demanded what he had first about him?

Numerical Solution.

- 1 For what he first had put x ,
- 2 Then by borrowing as much, he had $2x$,
- 3 And when he had spent 12 Pence, had $2x - 12$.
- 4 Then by borrowing the same, had $4x - 24$,
- 5 And by spending 12 Pence, had $4x - 36$;
- 6 Then, by borrowing the same, had $8x - 72$,
- 7 And by spending 12 Pence, had $8x - 84$.
- 8 Then by borrowing the same, had $16x - 168$,
- 9 Then by spending 12 Pence, had left $16x - 180$,
- 10 That is, had Nothing left. Whence (Q.)
 $16x - 180 = 0$,
- 11 That is, $16x = 180$.

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- 12 | Therefore $x = \frac{180}{16} = 11 \frac{4}{16}$
 | So that he had at first 11 d. $\frac{1}{4}$.

* PROBLEM XIX.

One being asked how old he was, answered thus:

If to the Number of my Age you add

The one Half of three Fourths, and 14 more,
The Number 58 will then be had:

What is my Age in Years above a Score?

Note, As the Question says, if you add 14 to his Age, it will make 58; so consequently without adding the 14 it will be 44; therefore working with 44, without adding 14, will be better than to work with 58 and 14 together, as you may observe in Problem 3d.

Numerical Solution.

- 1 | For his Age put x ,
 2 | Then by adding $\frac{1}{2}$ of $\frac{3}{4}$, that is, τ , it is $x + \frac{3x}{8}$.
 3 | This (Q.) = 44. Whence $x + \frac{3x}{8} = 44$.
 4 | This reduced $8x + 3x = 352$.
 5 | That is, $11x = 352$.
 6 | Therefore $x = \frac{352}{11} = 32$.
 | So that his Age was 32, 12 above a Score.

PROBLEM XX.

A Citizen riding his Rounds to receive Money due to him, came to a Place in which he had three Debtors, A, B, and C, but when he came to examine,

R

amine,

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amine, he found he had lost his Pocket-Book, in which was each Man's separate Bill. Then sending for them all to an Inn, he tells them the Accident; but they pretended they did not know what was due to him, having lost the Bills that came with the Parcels. The Gentleman thinking he had got some slippery Chaps to deal with, endeavoured all he could to save himself the Trouble of another Journey; and this he did from the following Data: He remembered very well that A's and B's Debt together made 13 £. 10 s. and A's and C's Debt together 31 £. 10 s. and B's and C's together made 37 £. 10 s. It is demanded what each Man's particular Debt was?

- 1 For A's Debt put x ,
- 2 Then as A's and B's together is 270 s. B's is $270 - x$,
- 3 Then as A's and C's is 630 s. C's alone is $630 - x$.
- 4 Now (Q) B's and C's should be 750 s. but is $900 - 2x$.
- 5 Whence this Equation $750 = 900 - 2x$.
- 6 By ϕ $2x = 900 - 750 = 150$.
- 7 And ϕ $750 - 150 = 600$.
- 8 That is, $2x = 150$.
- 9 Therefore $x = \frac{150}{2} = 75$ s. A's
- 10 And by 2d Step, $270 - x = 195$, B's.
- 11 And by 3d Step, $630 - x = 555$, C's.

So that A's Debt was $\begin{matrix} \text{£.} & \text{s.} \\ 3 & 15 \end{matrix}$
 B's $\begin{matrix} \text{£.} & \text{s.} \\ 9 & 15 \end{matrix}$
 C's $\begin{matrix} \text{£.} & \text{s.} \\ 27 & 15 \end{matrix}$

* P. R. O.

PROBLEM XXI.

A Highwayman robb'd a Gentleman of a certain Sum of Money, but being seen by 3 Men, A, B, and C, they pursued and took him; but he promised to make them a handsome Present if they would let him go; to which they agreed. To A he gave $\frac{1}{2}$, and A return'd him back 6 £. To B he gave $\frac{1}{3}$ of what was left, who return'd him 4 £. And to C he gave $\frac{1}{4}$ of what he had then left, who return'd him back 2 £. And after he had rode off, and came to tell his Money, he found he had given $\frac{2}{3}$ of it away: I demand the Sum he took from the Gentleman?

Numerical Solution.

- 1 For what he took in Pounds put x ,
- 2 Then by giving $\frac{1}{2}$ he had the same left, $\frac{x}{2}$,
- 3 And by A's returning him 6, he had $\frac{x}{2} + 6$
- 4 This he gave B $\frac{1}{3}$ of, viz. $\frac{x}{6} + 2$,
- 5 And consequently had himself $\frac{2}{3}$, viz. $\frac{2x}{6} + 4$,
or $\frac{x}{3} + 4$,
- 6 And by B's returning 4, had $\frac{x}{3} + 8$.
- 7 This he gave C $\frac{1}{4}$ of, viz. $\frac{x}{12} + 2$,
- 8 And consequently had in Hand $\frac{3}{4}$, viz. $\frac{3x}{12} + 6$,
- 9 And by C's returning 2, had $\frac{3x}{12} + 8$, or $\frac{x}{4} + 8$.

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- 10 This (Q.) = $\frac{1}{3}$ of what he stole. Whence

$$\frac{x}{3} = \frac{x}{4} + 8.$$
- 11 Then multiplying by the Denominator 3,

$$x = \frac{3x}{4} + 24.$$
- 12 This $\times$ the Denominator 4, $4x = 3x + 96.$
- 13 Then ϕ $3x$, it will be $4x - 3x = 96,$
- 14 That is, $x = \text{£. } 96$ *Ans.*
 So that he stole $\text{£. } 96$; of which *A* had 42,
B 14, and *C* $\text{£. } 8$, which together make $\text{£. } 64$
 $= \frac{2}{3}$ of 96.

PROBLEM XXII.

One being asked how many Children he had living, answered 3 Times as many as he had buried; and being asked how many that was, said, that if the Number he had lost was multiplied by the $\frac{1}{3}$ Part of what remained, it would be equal to the Number he had at first: I demand how many he had lost, and how many he had left?

Numerical Solution.

- 1 For what he had lost put x ,
- 2 Then will he have left $3x$,
- 3 And consequently had at first $4x$.
- 4 Now $\frac{1}{3}$ of what was left is $\frac{3x}{9}$, or $\frac{x}{3}$.
- 5 This multiplied by the lost, viz. x , is $\frac{xx}{3}$.
- 6 This (Q.) equals his Number at first. Whence

$$\frac{xx}{3} = 4x.$$
- 7 This reduced, $xx = 12x.$

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- 8 | Then dividing by x , — — $x = 12$, lost.
- 9 | And by 2d Step, $3x = 36$, left.
- 10 | Their Sum by 3d Step is $4x = 48$, at first.

PROBLEM XXIII.

One being ask'd how many Teeth he had, to avoid a direct Answer said, that he had lost the $\frac{1}{8}$ Part of what he then had, and being ask'd how many that was, said, that if what he had lost were multiplied into the $\frac{1}{4}$ of what he had left, and the Square of what he had lost was added to that Product, it would be equal to the Number he had at first. Or otherwise, if what he had lost were multiplied by $\frac{1}{2}$ of what he had left, it would make just $\frac{2}{3}$ of the Number he had at first: I demand how many he had lost, and how many he had left?

Numerical Solution.

- 1 | For the Number lost put x ,
 - 2 | Then as those left are 8 Times as many, $8x$,
 - 3 | These two added make the Number at first $9x$.
 - 4 | Now $\frac{1}{4}$ of the Number left is $2x$,
 - 5 | This multiplied by those lost, make $2xx$,
 - 6 | To which add the Square of those lost, it is $2xx + xx$.
 - 7 | These (Q.) are equal to the Number at first.
 - 8 | Then dividing each Side by x , $2x + x = 9$.
 - 9 | That is $3x = 9$.
 - 10 | Therefore $x = 3$, lost.
 - 11 | And by 2d Step, $8x = 24$, left.
 - 12 | And by 3d Step, $9x = 27$, at first.
- Which you may prove according to the Teror of the Question.

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PROBLEM XXIV.

An Usurer put out a certain Sum of Money at 5 £. per Cent. per Annum, which in 16 Years wanted exact 11 Guineas of the Principal itself: I demand what the Principal was?

Numerical Solution.

- 1 For the Principal put x ,
- 2 Then say, If 100 be 5, what will x be? *Ans.* $\frac{5x}{100}$.
- 3 This being 1 Year's Interest, 16 Years, is $\frac{80x}{100}$, or $\frac{8x}{10}$, or $\frac{4x}{5}$.
- 4 This should (Q.) be equal to the Principal less 11 Guineas, or 231 Shillings. Whence,

$$x - 231 = \frac{4x}{5}$$
- 5 This being reduced, $5x - 1155 = 4x$,
- 6 Then ϕ $4x$, — — — $5x - 4x = 1155$,
- 7 That is, $x = 1155s. = \text{£. } 57, 15s.$
 So that the Principal was $\text{£. } 57, 15s.$ which in 16 Years, at 5 per Cent. amounts to $\text{£. } 46. 4s.$ which wants 11 Guineas of the Principal.

PROBLEM XXV.

An Usurer put out 135 £. in 2 Parcels, one at the Rate of 5 per Cent. per Annum, and the other at 6 per Cent. which amounted in 15 Years Time to the Principal itself wanting 30 £. I demand the Parcel he put out at 5, and the Parcel he put out at 6 per Cent.

Numerical

Numerical Solution.

- 1 For the Parcel at 5 per Cent. put x ,
- 2 Then will that at 6 per Cent. be $135 - x$,
- 3 Then, by the Rule of 3, one Year's Interest
of x is $\frac{5x}{100}$,
- 4 And by the same Rule, one Year's of $135 - x$ is
 $\frac{810 - 6x}{100}$,
- 5 The Sum of the Interest of both for 1 Year is
 $\frac{5x}{100} + \frac{810 - 6x}{100}$.
- 6 Then because there is $5x$ and $-6x$, it will be
 $\frac{810 - x}{100}$.
- 7 This being 1 Year's Interest of both, 15 Years
will be $\frac{12150 - 15x}{100}$.
- 8 This (Q.) is equal to the Principal except £. 30.
Whence $105 = \frac{12150 - 15x}{100}$.
- 9 This $\times$ the Denominator 100, is
 $10500 = 12150 - 15x$.
- 10 Then $0 - 15x$ and 10500,
 $15x = 12150 - 10500$,
- 11 That is, $15x = 1650$.
- 12 Therefore, $x = \frac{1650}{15} = \text{£. } 110$.
- 13 And by 2d Step, $135 - x = \text{£. } 25$.
So that he put out £. 110 at 5, and £. 25
at 6 per Cent. the Interest of which in 15 Years
amounts to £. $105 = 135 - 30$.

P R O-

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PROBLEM XXVI.

It is required to pay 100 £. in 100 Pieces, viz. some to be 15 Shillings, and others 22 Shillings and 6 Pence each: I demand how many there must be of each Sort?

Numerical Solution.

- | | |
|----|--|
| 1 | Put for the 15s. Pieces x , |
| 2 | Then will those at 22s. 6d. be $100 - x$, |
| 3 | Now x Pieces, at 15s. or 180d. each is $180x$, |
| 4 | And $100 - x$ Pieces at 22s. 6d. or 270d. each, is $27000 - 270x$. |
| 5 | The Sum of these two is $27000 - 90x$. |
| 6 | This (Q.) = £. 100, or 24000d. Whence,
$24000 = 27000 - 90x$. |
| 7 | Then $\phi - 90x$, $90x + 24000 = 27000$ |
| 8 | Then $\phi 24000$, $90x = 27000 - 24000$, |
| 9 | That is, $90x = 3000$, |
| 10 | That is, $9x = 300$. |
| 11 | Therefore, $x = \frac{300}{9} = 33 \frac{1}{3}$. |
| 12 | And by 2d Step, $100 - x = 66 \frac{2}{3}$.
So that there was $33 \frac{1}{3}$ Pieces, at 15s. = £. 25
And $66 \frac{2}{3}$ Pieces, at 22s. 6d. = £. 75 |

PROBLEM XXVII.

A Vintner has two Vessels full of Wine, equal alike in Quantity, but of different Quality; the worst Sort is worth 240 Crowns, and the best 300: Now he has another Cask or Vessel of the same Size, which he intends to fill out of these two, that when full may be worth 260 Crowns: How much of each must he take?

Numerical

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Numerical Solution.

- 1 For what he must take of the worst Sort put x ,
- 2 Then as both make but one Vessel, the best will be $1 - x$.
- 3 Now if 1 full Vessel be 240 Crowns, x of a Vessel is $\frac{240x}{1}$.
- 4 And by the same Rule, $1 - x$ of a Vessel is $\frac{300 - 300x}{1}$.
- 5 The Sum of the Numerators of the 3^d and 4th Step is $300 - 60x$.
- 6 This (Q.) equal to the mean Price, 260 Crowns.
Whence this Equation, $260 = 300 - 60x$.
- 7 Then ϕ 260 and $-60x$, $60x = 300 - 260$.
- 8 That is, $60x = 40$.
- 9 Then must $6x = 4$. Therefore,
- 10 $x = \frac{4}{6} = \frac{2}{3}$. And by 2^d Step,
- 11 $1 - x = \frac{1}{3}$.

PROOF.

So that he must take $\frac{2}{3}$ of the worst, and $\frac{1}{3}$ of the best Sort.

For if 1 Vessel be 240 Crowns, $\frac{2}{3}$ will be 160,

And if 1 Vessel be 300, then $\frac{1}{3}$ will be 100,

Sum 260 = M. Price.

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PROBLEM XXVIII.

Two Men, A and B, set out from a certain Place, the one goes 21 Miles in 15 Hours, and 8 Hours after he set out, B begins to travel, and goes at the Rate of 15 Miles in 9 Hours: I demand how long it will be before B overtakes A, and how far they will both have travelled?

Numerical Solution.

- 1 For the Hours A travelled put x ,
- 2 Then will B travel $x - 8$.
- 3 Then, If 15 Hours be 21 Miles, what will x be

$$\text{Ans. } \frac{21x}{15}$$
- 4 And, If 9 Hours be 15 Miles, $x - 8$ will be

$$\frac{15x - 120}{9}$$
- 5 Now seeing that after B overtakes A, the Distance they travelled were both alike, therefore

$$\frac{15x - 120}{9} = \frac{21x}{15}$$
- 6 This reduced first by 9, is $15x - 120 = \frac{189x}{15}$
- 7 This $\times 15$, is $225x - 1800 = 189x$.
- 8 Then $\phi 189x$, $225x - 189x = 1800$,
- 9 That is, $36x = 1800$,
- 10 That is $x = \frac{1800}{36} = 50$ Hours, A.
- 11 And by 2d Step, $x - 8 = 42$ Hours, B,
- 12 Now A goes by 3d Step $\frac{21x}{15}$, that is,

$$70 \text{ Miles} = \frac{15x - 120}{9} = 70, B.$$

PRO-

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PROBLEM XXIX. *

What Number is that that $\frac{3}{4}$ of it more 12 is equal to $\frac{2}{3}$ of it more 14?

- 1 For the Number sought put x ,
- 2 Then $\frac{3}{4} + 12$, is $\frac{3x}{4} + 12$,
- 3 And $\frac{2}{3} + 14$, is $\frac{2x}{3} + 14$.
- 4 These (Q.) are equal.
Whence $\frac{3x}{4} + 12 = \frac{2x}{3} + 14$.
- 5 This reduced first, is $3x + 48 = \frac{8x}{3} + 56$,
- 6 That is, being again reduced,
 $9x + 144 = 8x + 168$.
- 7 Then $9x$ and $+144$, $9x - 8x = 168 - 144$,
- 8 That is, $x = 168 - 144 = 24$ *Ans.*
And this is proved at large in *Dial. 8. Sect. 4. Ex. 5.*

PROBLEM XXX.

Four Highwaymen, A, B, C, and D, robb'd a Gentleman upon the Road of 475 £. and going to an Inn to part the Money, which they had laid upon the Table, Words arose, and every one snatch'd up what he could; after which, upon telling each one his Money, it was found, that if to what A snatch'd up were added 4 £. and from B's were taken 4, and C's multiplied by 4, and D's divided by 4, it would produce one and the same Number of Pounds: It is demanded what each snatch'd up?

✶ First,

* See the Note in the Proof of Example 5, in Multiplication of Equations.

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First, Suppose *A* snatch'd x Pounds, then having 4 added, it would be $x + 4$. Now since by subtracting 4 from *B*'s, his would be equal to *A*'s, he must then have $x + 8$; and since by multiplying *C*'s by 4, he would have the same, he must of Course snatch $\frac{x}{4} + 1$, which $\times 4$, produces $x + 4$; and as *D*'s is to be the same if divided by 4, he must snatch $4x + 16$.

Numerical Solution.

- | | | |
|----|--------------------------------------|---|
| 1 | For what <i>A</i> snatch'd put x , | |
| 2 | Then will <i>B</i> 's be as above | $x + 8$, |
| 3 | <i>C</i> 's | $\frac{x}{4} + 1$, |
| 4 | <i>D</i> 's | $4x + 16$, |
| 5 | The Sum is | $6x + \frac{x}{4} + 25$. |
| 6 | This (Q.) is equal to the Robbery. | |
| | Whence, | $6x + \frac{x}{4} + 25 = 475$. |
| 7 | This reduced, | $24x + x = 100 + 1900$. |
| 8 | Then ϕ 100, | $25x = 1900 - 100 = 1800$. |
| 9 | Therefore, | $x = \frac{1800}{25} = 72$, <i>A</i> , |
| 10 | And by 2d Step, | $x + 8 = 80$, <i>B</i> , |
| 11 | And by the 3d, | $\frac{x}{4} + 1 = 19$, <i>C</i> , |
| 12 | And by the 4th, | $4x + 16 = 304$, <i>D</i> , |
| | | Sum 475. |

Tyr. This is a hard Question I think; at least I thought so at first reading.

Phi

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Phi. It may be so; and yet in other Words it may be easy; for it is no other than this: What 4 Numbers must 475 be divided into, so that the 1st having 4 added to it, the 2^d 4 taken from it, the 3^d multiplied by 4, and the 4th divided by 4, may be all equal? Which Numbers are as before, viz. 72, 80, 19, and 304; which you may prove.

PROBLEM XXXI.

Two Graziers, A and B, coming from a Fair, were met by two Highwaymen, who robbed A of 25 £. 10s. and B of 7 £. 10s. but upon their complaining that they had a great many Miles to ride, and Nothing to bear their Charges, he that robbed A return'd him a certain Sum, and so did the other to B. Now after A come to tell B what he had left, and B come to tell A, it was discovered that they robbed A of 3 Times as much as B, and left B $\frac{1}{4}$ of what they left A: I demand what each was really robbed of?

Numerical Solution.

- 1 For what they took from B put x ,
- 2 Then will what they took from A be $3x$
- 3 Now B had £. 7, 10s. or 150s. at first,
But now, $\underline{\hspace{2cm}} \quad \underline{\hspace{2cm}} \quad 150 - x$ left,
- 4 Also A had £. 25, 10s. or 510s. at first,
But now, $\underline{\hspace{2cm}} \quad \underline{\hspace{2cm}} \quad 510 - 3x$ left.
- 5 This (Q.) should be 5 Times what B had left.
Whence, $510 - 3x = 5(150 - x)$
- 6 Then $510 - 3x = 750 - 5x$
- 7 Then $510 - 3x = 750 - 5x$
- 8 That is, $2x = 240$
- 9 Therefore, $x = \frac{240}{2} = 120$ s. B.

S

| 10 |

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10 | And by the 2d Step, $3x = 360$, A .
 | So that they took £. 18 from A , and 6 from B .

PROBLEM XXXII.

A General of an Army had a certain Number of Men, which he intended to place in a square Battalia, but disposing of them in Rank and File, found he had 90 Men to spare; now thinking to get these in also, he enlarged his Square to one Man more in Rank and File, but then found he wanted 39 Men to complete the Square: What Number of Men had he, and how many stood in Rank and File?

Numerical Solution.

1		For the N ^o . that stood in Rank and File put x ,
2		Then will the Square of these be xx ,
3		But having 90 to spare, he had $xx + 90$.
4		Now encreasing Rank and File by 1, the Side is $x + 1$,
5		The Square of which is $xx + 2x + 1$,
6		From this take $xx + 90$, there remains $2x - 89$.
7		This (Q.) is equal to 39.
8		Whence, $2x - 89 = 39$.
9		Then $2x = 39 + 89$,
10		That is $2x = 128$.
10		Therefore, $x = 64$ a Side.

PROOF.

There were 64 Men in Rank, and 64 in File, and $64 \times 64 = 4096$, to which add 90, it is 4186. But had 65 Men been the Side of the Square, there would have been 4225 Men, which is 39 more than 4186, Q. E. D.

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* PROBLEM XXXIII.

There is a Vessel (partly) empty in which are 20 Gallons of Wine, worth 8 s. per Gallon; now if it be filled up with Water, the Wine and the Water together will be worth 6 s. a Gallon, and the Whole worth the same Money as when it was all Wine; I demand then what the Vessel holds when full, or, which is the same, how many Gallons of Water will fill it up?

Numerical Solution.

1 For what the Vessel wants in Gallons put x ,
 2 Now 20 Gallons of Wine, at 8 s. is 160 s.
 3 Now when the Vessel is filled up, the Whole will fetch the same as the Whole of the Wine did, viz. 160 s.

4 Then say, If $20 + x$ be 160 s. what is x Gallon? $\text{Ans. } \frac{160}{20+x}$

5 This (Q.) is equal to 6 s. Whence, $6 = \frac{160}{20+x}$

6 That is, $120 + 6x = 160$

7 Then $6x = 160 - 120 = 40$

8 Therefore $x = \frac{40}{6} = 6 \frac{2}{3}$

So that there wanted $6 \frac{2}{3}$ of Water to fill it:

To which add Wine 20 Gallons, it is $26 \frac{2}{3}$ Gallons the Contents.

PROOF.

20 Gallons of Wine, at 8 s. = 8 0

And $26 \frac{2}{3}$ of mixt, at 6 s. = 8 0

* PROBLEM XXXIV.

Vertruvius, (Lib. ix. Chap. 3.) informs us, That King Hiero being obliged by Vow to make a Present of a Crown of pure Gold, weighing 100 lb. gave Orders for such an one to be made; but being told that the Goldsmith had secreted Part of the Gold, and put to the Crown the same Weight of Silver, he sent for the famous Archimides of Syracuse, to whom he recommended the Discovery of the Fraud: It is demanded how Archimides discovered the Cheat, and how many Pounds of Silver the Goldsmith had put into the Crown?

Since it is proved by Experiment that a Mass of pure Gold will possess less Space than a Quantity of Silver of the same Weight, it will be easy to conceive that a mixt Mass of Silver will possess, or take up a Space between them. *Virtruvius* therefore caused two Masses to be made of equal Weight with the Crown, one all of pure Gold, and the other all Silver; then having a Vessel filled to the Brim with Water, he caused the Crown to be immersed, carefully reserving the Water which flowed over. And thus he did with the Mass of Gold and Silver, reserving each Time the Water which flowed over the Vessel; by which Means he very exactly told *Hiero* how much Gold was secreted.

Now let us suppose, that by immersing the Mass of Gold only, there was emitted 60 lb of Water, and by the Mass of Silver 90 lb, and by the Crown 64 lb. Then,

Numerical Solution.

1		For the Quantity of Silver in the Crown put x ,	
2		Then must the Gold	100 — x

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- 3 Now if 100 lb. of Gold be 60 lb. Water,
 $100 - x$ is $\frac{6000 + 30x}{100}$.
- 4 Again, If 100 lb. of Silver be 90 lb. Water,
 x is $\frac{90x}{100}$.
- 5 The Sum of the two is $\frac{6000 + 30x}{100}$.
- 6 This ought then to be equal to the Pounds
of Water emitted by the Crown, viz. 64.
- Whence, $\frac{6000 + 30x}{100} = 64$.
- 7 This reduced, $6000 + 30x = 6400$,
- 8 That is, $30x = 6400 - 6000$,
- 9 That is, $30x = 400$
- 10 That is, $3x = 40$
- 11 Therefore, $x = \frac{40}{3} = 13 \frac{1}{3}$.
- So that the Goldsmith had mixt $13 \frac{1}{3}$ lb. of
Silver in the Crown.

N. B. But there may be an infinite Number of
Answers produced, according to the Variation of
the Question, and the Crown will be more or less
adulterated, according to the Proportion of the Wa-
ter emitted by the Mass of Gold and the mixt Mass;
for the less their Difference, the less the Adultera-
tion.

Note also, That this and such like Questions may
be done by knowing the specific Gravity of each
Body; that is, weighed separately in Air and Water,
and their Proportions will hold good in the same
Manner as above, and is more modernly practised.

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PROBLEM XXXV.

What Number is that whose $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ exceeds the Whole by 240?

Numerical Solution.

- 1 For the Number put x ,
- 2 Then will its $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, be $\frac{x}{2}$, $\frac{x}{3}$, $\frac{x}{4}$.
- 3 These reduced to a C. D. will be $\frac{12x + 8x + 6x}{24}$
or rather, $\frac{26x}{24} = \frac{13x}{12}$.
- 4 Whence (Q.) this Equation, $\frac{13x}{12} = x + 240$.
- 5 This reduced, $\frac{13x}{12} = 12x + 2880$.
- 6 And ϕ $12x$, $\frac{13x}{12} - 12x = 2880$,
- 7 That is, $\frac{13x}{12} - 12x = 2880$, the Number.

PROBLEM XXXVI.

There is a Rod of Iron a Yard long, at the Ends of which hang 2 Weights, viz. one of 15, and the other of 1 lb. Weight: I demand the Point of the Rod where these two Weights will hang in Balance?

Numerical Solution.

- 1 For the Distance of the less Weight to the Point put x ,
- 2 Then will the Remainder be the Distance of the Greater, viz. $36 - x$ Inches.
- 3 Then will the Proportion be,
As $x : 36 - x :: 15 : 1$.
- 4 Multiplying Means and Ext. $1x = 540 - 15x$,
- 5 Then ϕ $15x$, $\frac{16x}{16} = \frac{540}{16}$.

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- 6 | Therefore, — $x = \frac{540}{16} = 33 \frac{1}{4}$ Less Wt.
 7 | And by 2d Step, $36 - x = 2 \frac{1}{4}$, Greater.
 So that the Point from the small Weight must be
 33 $\frac{1}{4}$ Inches, and the Distance of the 15 lb.
 Weight 15 Times less, viz. 2 $\frac{1}{4}$ from the End.

PROBLEM XXXVII.

Suppose a Rod of Iron to be equally divided into 150 equal Parts, and at the first Part or Division hangs a 4 lb. Weight, and on the last Division, or other End, a Weight of 4 Score and 16 Pounds: I demand the Point of the Rod where these two Weights will be in Equilibrio; or, which is the same, what Division of the Rod will be a Balance to both the Weights?

Numerical Solution.

- 1 | For the Difference of the greater Weight put x ,
 2 | Then will the less be $150 - x$.
 3 | Then. — As $x : 150 - x :: 4 : 96$.
 4 | By multiplying Means and Ex. $96x = 600 - 4x$.
 5 | Then $\phi - 4x$, — — $100x = 600$.
 6 | Therefore, $x = \frac{600}{100} = 6$, greater Weight.
 7 | And by 2d Step, $150 - x = 144$, Less Weight.
 So that the Point of Balance is 6 Parts from the
 greater Weight, and 144 from the Less: For
 $144 + 6 = 150$, the Whole.

☞ From hence also, if a Rod be divided into any Number of Parts, and one Weight be given, and the Point given in the Rod, the other Weight is easily found by Proportion, thus: As the Distance of one End is to the given Weight, so is the other Distance from the End to the required Weight.
 Thus,

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Thus, in the *Problem* before us, let the Weight 6 be given, and the Division 144 on which it is placed, and the whole Rod 150, as before, to find what Weight will balance it.

PROBLEM XXXVIII.

There is a Rod divided into 150 equal Parts, on which hangs at one End a 4 lb. Weight, and the Rod being placed or laid across any Thing at the 144th Division, I demand what Weight at the other End will be able to balance the 4 lb. Weight, to keep the Rod in Equilibrio?

This is done by the Help of the Rule of Proportion, either direct or inverse. For only observing the Distance the Point of Balance is from the given Weight, (which here is 144, and the Remainder 6) the Proportion is, As 6, the Remainder of Divisions from the less Weight, is to the Weight itself, so is the Distance of the less Weight from the Point of Balance to the greater Weight, &c. Thus,

As 6 : 4 :: 144 : 96. Or,

As 6 : 144 :: 4 : 96, &c.

By this Method may be proved, whether the *Steel-yard*, or any Beam or Pair of Scales belonging be good : For notwithstanding the vulgar Notion of Beams and Scales being true, because the *Brachium* hangs in Balance, it is evident, that in weighing large Quantities the Buyer may be sufficiently cheated, or the Seller may ignorantly cheat himself, and that more or less, in Proportion to the Make of the Beam.

Tyr. This is a Sort of a Paradox to me at present !

Nov.

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Nov. And to me likewise; I wish, therefore, *Philomathes*, you would explain it a little to me: For if it be so, how am I to choose a good Beam, or depend upon others to know whether they be true or false; for it is not to be supposed every one can prove it by Figures?

Phi. I grant it; but you may soon see the Truth of this by several Experiments*. However, I will tell you thus much, that in choosing your Beam, mind not altogether its hanging in Balance; but more particularly examine whether the Arms whereon the Scales hang be equi-distant from the Center; for should they not, you may depend it is not an honest Balance: And whenever you suspect any Pair of Scales, you may satisfy yourself by this vulgar Experiment only; change the Weights and Commodity to the contrary Scale, and if the Weight be as before it is right, otherwise false.

Nov. We are obliged to you for this easy Explanation.

Phi. I am not treating of *Mechanics* it is true; but I have said Something the more upon it, because it is more useful and necessary in Business than every common Question; and indeed, such Persons as deal in valuable Commodities in large Quantities, should be careful to examine Things of this Nature. But come, we will proceed to

PROBLEM XXXIX.

A Tradesman began the World with a certain Sum of Money, with which he bought a Stock of Goods, but by Misfortunes in Trade, he lost the first Year one Half the Value of his Stock, and 10 Shillings over; and

* See Dr. Desaguliers's *Experimental Philosophy*, Vol. 1. Plate 7, 8, 9, and 10.

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and also the second Year he lost Half his Stock, and 10 Shillings over: and thus he went on for 5 Years, losing Half the Value of the preceding Year's Stock, and 10 Shillings over: Now at the End of the fifth Year he left off Trade, and his Stock was worth but 50 £. 10 s. I demand the Value of his Stock at first?

Numerical Solution.

- 1 For the Value of his Stock at first in Pounds put x ,
- 2 Then he lost the first Year $\frac{x}{2} + \frac{1}{2}$ *,
- 3 And had left $\frac{x}{2} - \frac{1}{2}$, viz. $\frac{x-1}{2}$.
- 4 Then had he lost just $\frac{1}{2}$ this, the 2^d Year it would be $\frac{x-1}{4}$.
- 5 But he lost 10 Shillings more, therefore he had left $\frac{x-1}{4} - \frac{1}{2}$.
- 6 This reduced, first $\times$ the Denominator 4, then $\times 2$, is $\frac{2x-2-4}{1}$ or, $\frac{2x-6}{8}$.
- 7 This abbreviated, is the 2^d Years Stock left $\frac{x-3}{4}$.
- 8 Then by losing Half + 10 s. had left the 3^d Year, $\frac{x-3}{8} - \frac{1}{2}$.
- 9 This reduced, is $\frac{2x-6-8}{16}$ or, $\frac{2x-14}{16}$ or, $\frac{x-7}{8}$.

* Note, As 10 Shillings is Half of a £. the $\frac{1}{2}$ in the 2^d Step represents 10 Shillings, and saves a great deal of Trouble.

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- 10 The Half of this, less 10 Shillings is left the 4th Year, viz. $\frac{x-7}{16} - \frac{1}{2}$. Or,
- 11 This reduced, is $\frac{2x-14-16}{32}$ or, $\frac{2x-30}{32}$
or, $\frac{x-15}{16}$.
- 12 Then had he left Half this, less 10 Shillings the 5th Year, viz. $\frac{x-15}{32} - \frac{1}{2}$,
- 13 This reduc'd, is $\frac{2x-30-32}{64}$ or, $\frac{2x-62}{64}$
or, $\frac{x-31}{32}$.
- 14 Now this (Q) is equal £. 50, 10s. or, $50 \frac{1}{2}$.
Whence, $\frac{x-31}{32} = 50 \frac{1}{2}$
- 15 This reduced, first $\times$ Denominator 32,
 $x-31 = 1600 \frac{32}{2}$.
- 16 This $\times$ the Denominator 2, is
 $2x-62 = 3200 + 32$.
- 17 Then $\phi - 62$, $2x = 3200 + 32 + 62$.
- 18 Therefore, $x = \frac{3294}{2} = \text{£. } 1647$, his 1st Stock, which you may prove at Leisure.

PROBLEM XL.

What Number is that which if added severally to 3, 19, and 51, will make them 3 Proportionals?

Numerical Solution.

- 1 For the Number put x ,
- 2 Then by adding this to each Number, they are
 $x+3$, $x+19$, $x+51$.
- 3 Whence, by the Rule of Proportion,

As

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- As $x + 3 : x + 19 :: x + 19 : x + 51$.
 4 But multiplying *Means* and *Extremes*, you have
 $xx + 54x + 153 = xx + 38x + 361$
 5 Then by cancelling xx on both Sides,
 $54x + 153 = 38x + 361$.
 6 Then ϕ $38x$ and 153 , it is
 $54x - 38x = 361 - 153$,
 7 That is, $16x = 208$.
 8 Therefore, $x = \frac{208}{16} = 13$ *Ans*.

P R O O F.

Numbers	3	19	and	51
Add	13	13		13
	16	32		64

For $16 \times 64 = 32 \times 32$.

Do these Operations appear plain to you *Novitius*?

Nov. Very plain, Sir; I think I understand them all very well.

Tyr. I wish I could say so for my Part, for I must own at present I do not.

Phi. It is not to be expected so young a Learner as you, *Tyrunculus*, should be Master of these Things at once: If you understand the Work by reading it, that is sufficient at present, and in going through the *Problems* once more, you will, no Doubt, understand them; and I think I have given you a Variety of Examples enough.

Nov. I beg you would work a Question or two in plain *Trigonometry* before you conclude, if *simple Equations* will perform them.

Phi. Yes, there are many to be performed by *simple Equations* only: But really I have scarce Room

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to grant your Desire, having already added ten *Problems* more than I intended. However, to oblige you I will; but then I shall work them *literally*, for it will be too tedious to do them *numerically*.

PROBLEM XLI.

Suppose a Pole to stand upon an Horizontal Plane, 75 Feet clear from the Ground (or $x + c$); what Height from the Ground must it be cut or sawn off at, so that the Top of it may fall upon a Point 55 Feet from the Bottom of the Pole to the said Point on the Ground?

Literal Solution.

- 1 For the Height sought put x ,
- 2 The Square of which is xx ,
- 3 Then is the Square of b bb ,
- 4 The Sum of their Squares $xx + bb$,
- 5 The Remainder to 75 is $c - x$,
- 6 The Square of this is $cc - 2xc + xx$.
- 7 Whence this Equation, $xx + bb = cc - 2xc + xx$.
- 8 Cancel xx on both Sides, $bb = cc - 2xc$.
- 9 Then $\phi - 2xc$, it is $2xc + bb = cc$.
- 10 Then ϕbb , $2xc = cc - bb$.
- 11 Then must $x = \frac{cc - bb}{2c}$ that is, $x = 75 \times 75 - 55 \times 55$, divide by 75×2 , which is $= 17 \frac{1}{2}$ Feet *Ans.*

T

PRO-

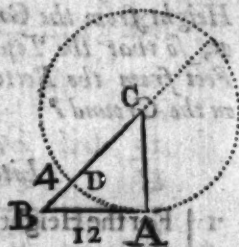
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PROBLEM XLII.

In the Triangle ABC is given the Base $AB = 12$, and the Segment, or Part of the Hypotenuse, BC , viz. $BD = 4$. Required the Sides AC and BC?

Literal Solution.

Let $a = 12 = AB$, and $b = 4 = BD$; and let $x = AC$. Now as AC and CD are equal, BC must be $= b + x$. Then by the 47th of Euclid, $BC^2 = AC^2 + AB^2$; but $BC^2 =$ the Square of $b + x$, viz. $bb + 2bx + xx$, and $aa = AB^2$



- 1 The Square of $b + x$ is $bb + 2bx + xx$.
- 2 This (Q.) is $=$ Square of x and a .
Whence, $bb + 2bx + xx = xx + aa$,
- 3 By cancelling xx on both Sides, $bb + 2bx = aa$,
- 4 Then ϕ bb , it will be $2bx = aa - bb$,
- 5 Therefore, $x = \frac{aa - bb}{2b} = \frac{12 \times 12 - 4 \times 4}{2 \times 4} = 16$
So that the Side CA or $CD = x = 16$. And
 $CD 16 + BD = 4 = 20 =$ Hypotenuse CB .

PROBLEM XLIII.

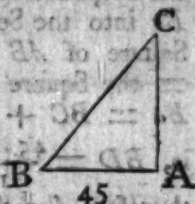
There is a rectangled Triangled ABC, whose Base $AB = 45$, and the Sum of the Hypotenuse and Cathetus $AC + BC = 135$: It is required to find the Sides AC and BC separately?

Literal

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Literal Solution.

Let $d = 135 = AC + BC$, and let $b = 45$, or BA , and put x for CA ; then $CB = d - x$. And seeing the Angle CAB is a right one, we have (by the 47th of 1 Euc.) $BC^2 = CA^2 + BA^2$; but $BC^2 =$ the Square of $d - x$, viz. $dd - 2dx + xx$. And $CA = xx$; and $BA = bb$. Therefore.



- 1 The Square $d - x$ is $dd - 2dx + xx$.
- 2 This (Q.) = to the Square of x and b .
Whence, $dd - 2dx + xx = xx + bb$.
- 3 Then cancelling xx , $dd - 2dx = bb$.
- 4 Then ϕbb , $dd - bb = 2dx$.
- 5 Therefore, $x = \frac{dd - bb}{2d}$, or, $\frac{d}{2} - \frac{bb}{2d} = 60$
 $= CA$. And $135 - 60 = 75 = BC$.

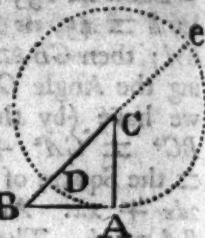
O R,

Let $x = BC$, and then will $AC = d - x$; and therefore (by the 47th of 1 Euc.) $BC^2 = AC^2 + AB^2$. Therefore, as $b = BA$, as above, it will be

- 1 $xx = dd - 2dx + xx + bb$,
- 2 $0 = dd - 2dx + bb$,
- 3 $2dx = dd + bb$,
- 4 $x = \frac{d}{2} + \frac{bb}{2d} = \frac{135}{2} + \frac{45 \times 45}{2 \times 135} = 75$ the
Hypotenuse BC . And $135 - 75 = 60 = AC$, as before.

T 2 Note,

Note, By the 36th of 3 *Euclid*, you may find the Sides *BC* and *AC* thus: Describe a Circle, making the Perpendicular the Radius; then is the Rectangle *Bc* into the Segment *BD* = the Square of *AB*; therefore *BD* is = the Square of *AB* divided by *Bc* = *BC* + *AC* = 135; that is, $BD = \frac{45 \times 45}{135} = 15$; and therefore (by the Figure and the preceding Work) *BC* + *AC*; that is, $\frac{135 - 15}{2} = 60$ *AC*, and $60 + 15 = 75$ *BC*.



Geometrically.

First, From any Scale of equal Parts make the Linè $AB = 45$, and at right Angles to it draw $Ae = 135$, the Sum of the Sides AC and BC . Then join Be with a right Line from e to B , and divide this in the Middle at b ; then let fall a Perpendicular from b upon the Line Ae , which will fall upon the Point C ; then having drawn the Line BC , the Triangle is compleated: And if you measure the Sides upon the same Scale, you will find $BC = 75$, and $AC = 60$, as above: For the Line bc being perpendicular to Be , and cutting it into two equal Parts; the Triangle BCE is *Isofcles*, by the 5th of 1 Euc. Therefore consequently $AC + BC = AC + Ce$. Q. E. D.



And thus, *Novitius*, I have done all that is in my Power to serve you and *Tyrunculus*; and I shall leave

7 Problems

ALGEBRAIC PROBLEMS. 209

7 Problems more, without their Operation, (to make up the Number 50) for your Practice; and I desire you would assist *Tyrunculus* in them, as I have assisted you in the others.

S E C T. II.

Here follows some more PROBLEMS, to exercise the young ALGEBRAIST.

P R O B L E M XLIV.

One hires a Farm containing 125 Acres of Ground, for which he gives 38 £. 5 s. the Land consists of 2 Sorts; for the better Sort he gave 7 s. 6 d. per Acre, and for the worst 3 s. 9 d. per Acre: I demand how many Acres there were of each Sort?

P R O B L E M XLV.

One lets out 60 £. in 2 Parcels, one at 5, and the other at 6 per Cent. which in 13 Years simple Interest wanted but 19 £. 7 s. 6 d of the Principal: I demand the Parcels?

P R O B L E M XLVI.

Three Drunkards, A B, and C, having each of them run up a separate Score at an Alehouse, agreed to go (under Pretence to drink) and rub all out; which was done accordingly: But the Landlord remembered very well, that A's and B's reckoning added together made 16 s. 10 d. $\frac{1}{2}$, and B's and C's 13 s. 3 d. $\frac{1}{4}$, and A's and C's 11 s. 5 d. $\frac{1}{2}$: He therefore craves your Assistance from hence, to tell him each Man's distinct Score?

T 3

P R O.

210 ALGEBRAIC PROBLEMS.

PROBLEM XLVII.

There are two Numbers whose Sum is 517, and the Quotient of the Greater by the Less is just 1000: I demand the Numbers?

PROBLEM XLVIII.

What Number is that, which, if added to 33, 209, and 561, will make them 3 Proportionals?

PROBLEM XLIX.

What Number is that whose $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{12}$, exceeds itself by 1?

PROBLEM L.

A Person dying, left in Cash 410 £. 10s. to his 4 Sons, A, B, C, and D, in such a Manner, that if A had had 4 £. 10s. more, and B 4 £. 10s. less, and C's were multiplied by 4 £. 10s. and D's divided by 4 £. 10s. it would produce one and the same Sum of Money: What was the Portion of each?

Nov. But pray why do you not insert their Answers?

Tyr. It would be some Help I think to the young Practitioner.

Phi. It would be indulging him you might say indeed, but I cannot see why the Questions are at all the harder to be done, without it be to such who from the Answers often guess out the Numbers; therefore I choose rather to omit the Answers: For if you do the Work right, it will prove itself; and to a diligent Learner it is all the same as if the Answers were before him; and I am sure it is a proper Exercise.

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ercise to qualify you for more difficult Things of this Sort.

Nov. You may depend upon it, Sir, I will do my Endeavour to find their Answers in a short Time.

Tyr. So will I, as soon as I am a little more perfect in the foregoing *Problems*.

Phi. You are right, *Tyrunculus*, for from a true Knowledge of them you will soon discover these also.

Nov. We are highly obliged to you, *Philomathes*, for these Favours.—Come, *Tyrunculus*, do you think of going?

Tyr. When you please, Sir.

Nov. Dear *Philomathes*, in accepting my hearty Thanks you will yet more oblige your humble Servant.

Tyr. Pray receive mine also.

Phi. I do; and you are not only welcome to these small Instructions, but I shall always be ready to serve you: Only let me persuade you (as far as Things of more Moment will allow of) to assist one another; for it is possible I may (by and by) instruct you in something of *Quadratic Equations*, because it is a Pleasure to me to see you delight thus in Figures. Those that have no Taste for this Sort of Learning indeed, are ignorant of the Satisfaction that it leaves; for what can be a greater Satisfaction to the Mind than *Certainty* itself, built upon the Foundation of unerring Principles? This made a noted Author say, that “*Algebra*, like *Logic* gives us a just Idea of the Nature of Things, shews us the true Way of reasoning, elevates the Mind to a proper Degree, and will not suffer it to dwell upon mean and base Trifles.”

And I could heartily wish that more of the growing Youth of this Age (especially such as can afford it)

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it) would (with you) give their Minds to the Study of some of the *Mathematical Sciences*, they being not only useful, but very diverting, and would certainly tend much more to their own private Good, and *that* of others, rather than the constant Perusal of such Books which daily vitiate the Mind, and corrupt the *Morals*. Thus we read, “*Xenophon* commended the *Persians* for their careful and prudent Education of their Children, who made them study only such Authors as treated of Learning and Morality; but would not suffer them to effeminate themselves with idle and amorous Tales, knowing well, and wisely discerning, *that there needed no Weight to be added to the Bias of corrupt Nature.*”

N. B. Page 173, Line 23, for $\frac{22}{2}$ read $\frac{12}{2}$.





AN

APPENDIX

CONTAINING

Some necessary INSTRUCTIONS

In the RUDIMENTS of

QUADRATIC EQUATIONS,

VIZ.

I. INVOLUTION, or the Method of raising
POWERS or involving QUANTITIES.

II. The RESOLUTION of a SQUARE raised
from a BENOMINAL, and how to compleat
the SQUARE.

III. Of EVOLUTION or extracting ROOTS.



APPENDIX

In the Rudiments of

QUADRATIC EQUATIONS

VIZ.

I. Invention, of the Method of raising Powers or involving Quantities.

II. The Resolution of Equations raised from the preceding, and the Completing the Square.

III. Of Evolution or extracting Roots.



APPENDIX, &c.

Or a DIALOGUE between

PHILOMATHES and NOVITIUS

CONTAINING

Some necessary INSTRUCTIONS

IN

QUADRATIC EQUATIONS.

PHILOMATHES *calls upon* NOVITIUS *to know*
what Improvement he has made in SIMPLE EQUA-
TIONS.

☉ *The Sign of INVOLUTION.*

∞ *The Sign of EVOLUTION.*

√ *The Sign of a SURD or IRRATIONALITY.*

Phi.  YRUNCULUS your Servant.

Nov. Sir, I am heartily glad to
see you.

Phi. You remember I promised
to give you some Notion of Qua-
dratics, which I intended to have
done before, but that Business of greater Moment
has continually called for my Attention: And though
I am

I am now come according to Promise; yet my Visit will be but short: And before I begin with you let me know whether you are pretty perfect in what I have shewn you before.

Nov. That I assure you I am.

Phi. We will proceed to the Point in Hand then.

Nov. Pray what does the Knowledge of *Quadratics* depend upon?

Phi. The Knowledge of *Quadratic Equations* depend upon these 4 Things.

1st. The Method of raising Powers from a single Quantity.

2^{dly}. The Resolution of a Square raised from a *Binominal* or *Residual*, and how to complete the Square when two Members only are given. And

3^{dly}. The Way or Method of extracting the Roots.

The first two of these are comprehended under the Name of *Involution*, or the Method of involving Quantities from any given Root.

Of raising Powers from a single Quantity.

RULE.

Multiply the given Quantity into itself you have the Square, to which join the said Quantity you have the Cube or third Power, &c. &c.

EXAMPLE I.

Let x the Root be involved to the 2, 3, 4 and 5th Power.

x Root.

xx Square or 2^d. Power.

xxx Cube or 3^d. Power.

$xxxx$ Biquadrate or 4th. Power.

$xxxxx$ Sur-solid or 5th Power.

Nov.

APPENDIX. 217

Nov. This is so plain, more Examples are needed: But what do you mean by a *Binominal* and *Risidual* Root?

Phi. A *Binominal* is a compound Quantity consisting of two Parts, as $x + b$ or $x + \frac{b}{4}$ connected together by the Signs more $+$ and less $-$ or $+$ or $-$ and also $x - b$ or $x - \frac{b}{4}$. Now these two Parts multiplied by themselves (that is squared) will always produce 3 Members, the first and last of which will be perfect Squares of the Root itself, and will always be *affirmative*; and the middle Part or Member is made by the double Rectangle of the Parts of which the *Binominal* is composed, and this middle Part will be sometimes *Affirmative*, and sometimes *Negative*: *Affirmative* when both are *Affirmative* or *Negative*, as $x + b$ or $-x - b$; and *Negative* when one of the Parts are *Negative*, as $x - b$ *; do you understand me?

Nov. Yes very well, except it be the double Rectangle you talk of.

Phi. This I shall satisfy you about presently, under Observation the First: In the mean Time, we will give you an Example.

2dly. The Resolution of a Square raised from a *Binominal* or *Risidual*, and how to complete the Square when two Members only are given.

U

Ex-

* Note, A *Binominal* is called a *Risidual* Root when one Part is negative as $x - b$.

EXAMPLE 2.

Let $x + b$ a *Binominal* be raised or involved to the third Power.

$$\begin{array}{r} x + b \\ x + b \end{array}$$

$$\begin{array}{r} xx + xb \\ \quad\quad xb + bb \\ \hline \end{array}$$

$xx + 2xb + bb$ the Square. This $\times x + b$
 $x + b$

$$\begin{array}{r} xxx + 2xxb + xbb \\ \quad xxb + 2xbb + bbb \end{array}$$

$$xxx + 3xxb + 3xbb + bbb \text{ Cube.}$$

EXAMPLE 3.

Let $x - b$ a *Risidual* be raised or involved to the third Power.

$$x = b$$

$$xx - xb$$
$$- xb + bb$$

$$xx - 2xb + bb \text{ Square } \times x - b$$

$$\begin{array}{r} xxx - 2xxb + xbb \\ \hline - xxb + 2xbb - bbb \end{array}$$

$xxx - 3xxb + 3xbb - bbb$ Cube.

Nov.

Nov. I understand you very well ; and I also perceive that in the *Binominal* the Answer is affirmative in all the Quantities ; but in the *Risidual* they alternately change.

Pbi. You say right ; and you see that it is Nothing but common Multiplication ; and if there were Fractions the Work is the same.

EXAMPLE 4.

Thus, $\frac{x}{2}$ squared is $\frac{xx}{4}$, for only multiply the Denominators and the Numerators together as in Multiplication of Algebraic Fractions, so also $\frac{x}{4}$

squared $\frac{xx}{16}$; and $\frac{x}{4} + \frac{b}{2}$ squared is $\frac{xb}{8}$; and $x + \frac{a}{2}$

a *Binominal* is $xx + \frac{2ax}{2} + \frac{aa}{4}$, or $xx + ax + \frac{aa}{4}$.

And lastly, $x - \frac{a}{2}$ will also be $xx - \frac{2ax}{2} + \frac{aa}{4}$, or

$xx - ax + \frac{aa}{4}$. For 2 in the middle Term being

common to the Numerator and Denominator I expunge it, and take the Numerator ax only.

Tyt. I thank you kind Sir, for your demonstrating it so plainly.

Pbi. I presume then, as you know how to involve any Root, you also know the third Thing, that is, how to complete the Square.

Nov. That I do not, nor do I know altogether at present what you mean.

Pbi. I own as I said before, that it is not so easily known by a Learner ; therefore I readily excuse it, because Authors in general have taken no Notice of this, though so necessary.

Of compleating the Square when but two Members are given.

I have already told you, that any compound Quantity, whether *Binominal* or *Risidual*, when squared, will consist of these Members; the Middle of which will be sometimes *affirmative*, and sometimes *negative*: I also told you, that when they are both perfect Squares you may easily know it, as $xx + bb$ or $xx + 16$; to compleat the Square of which will easily appear as follows.

OBSERV. I.

When any compound Quantity as $xx + bb$ or $xx - bb$ wants to be compleated, that is, wants the middle Part or Member, (for remember I told you it consists of 3) then take the Root of each Part, viz. x and b , and multiply them together, which is xb or $-xb$, this is what we call the Rectangle, the Double of which is $2xb$ or $-2xb$, either of these put between the other two will compleat the Square.

Nov. This is easy indeed if I take it right; for suppose $xx + bb$ were to be compleated, the Root of xx is x the Root of bb is b ; now $x \times b = xb$ the single Rectangle, therefore $2xb$ is the double Rectangle, which placed between the other two Members will be $xx + 2xb + bb$, or $xx - 2xb + bb$. But how am I to proceed when there are Co-efficients?

Phi. The very same: For suppose the Root $8x - 2$ were to be squared or compleated, here $8x \times 8x = 64x$ the first Term, and $-2 \times -2 = 4$ the third Term. Now $8x \times -2 = -16x$ the single Rectangle, therefore $-32x$ is the double Rectangle or middle Term, so the Square compleated is $64x - 32x + 4$.

Numerical

Numerical Demonstration.

Suppose they were in Numbers only, you will see the middle Term is always made up with the double Rectangle of the Parts. For let any Number (suppose 16) be divided into any two Parts, as 12 and 4. To compleat the Square $4 \times 12 = 48$, which doubled, is 96 the middle Term, so is the Square compleated, viz. $144 + 96 + 16$; and if you make the Binominal $x + b = 12 + 4$, you will in course have $xx + 2xb + bb = 144 + 96 + 16$.

Now. I like this very well; but pray how do you compleat the Square? when of the two given Members only, the first is a Square, and the other any Quantity or Number proposed at random.

Phi. To be sure this cannot be done in many Cases, by taking the double Rectangle of the Parts as before directed, because the Parts consist not of two pure or perfect Squares; but still, *Tyrunculus*, we shall put you into an easy Way of doing it at once, I'll warrant you, by the third Thing proposed, namely,

OBSERV. 2.

When any two Quantities are proposed, whereof the first is a Square, whether they have Co-efficients or not, or whether the second Member be a Fraction or not, you may find the third Member and compleat the Square of two such Quantities by this Rule only.

Another general Rule to complete the Square of any two Quantities, one whereof is a perfect Square.

RULE.

Take half the Co-efficient of the second Member, and the Square thereof shall be the third Member,

U 3

which

which will compleat the Square of the said two given Members.

Tyr. What do you say this will do, though I propose any Quantities or Fractions at Random?

Phi. Yes, provided your first Member be a perfect Square, and the second has the Root of that Square found in it.

Tyr. Give me a few Examples.

Phi. I will.

EXAMPLE 1.

Suppose $xx + 8x$ were to be compleated here, half the Co-efficient of the second Member, viz. 8 is to this squared is 16 which will be the third Term. $xx + 8x$ therefore when compleated, will be $xx + 8x + 16$, the Root of which is $x + 4$, for $x + 4 \times x + 4 = xx + 8x + 16$.

EXAMPLE 2.

Let $xx + 14x$ be compleated.

Here half of 14 is 7, this squared is 49; so that $xx + 14x$ when compleated is $xx + 14x + 49$.

Nov. Very easy indeed, and very pretty.

Phi. Notwithstanding this you shall very rarely meet with it in Authors.

Nov. I know it; but pray suppose the second Member have an odd Number or Co-efficient, or suppose it to have Fractions how then?

Phi. The very same as before.

EXAMPLE 3.

Let $xx + 5x$ be compleated.

Nov. I am at a Loss at present indeed.

Phi. Surely not, *Tyrunculus!* why is not the Half of 5 expressed $\frac{5}{2}$?

Nov.

Nov. I ask Pardon, it is so, and the Square of $\frac{2}{3}$ is $\frac{4}{9}$, is it not?

Phi. Be sure it is.

Nov. Then I perceive $xx + 5x$ when compleated, will be $xx + 5x + \frac{25}{4}$. And by the same Rule $xx - \frac{2x}{5}$ will be $xx - \frac{2x}{5} - \frac{1}{25}$ for the $\frac{1}{5}$ of $\frac{2}{5}$ is $\frac{2}{25}$ and the Square of $\frac{1}{5}$ is $\frac{1}{25}$, shew me one or two literally.

EXAMPLE 5.

Suppose $xx + bx$ be given, what will the third Member be to compleat the Square?

Here the Co-efficients of the second Member is b , Half of which is $\frac{b}{2}$, which squared, is $\frac{bb}{4}$, so that $xx + bx$ when compleated $xx + bx + \frac{bb}{4}$ See Page 219, Example 4.

EXAMPLE 6.

$xx + \frac{bx}{a}$ when compleated, is $xx + \frac{bx}{a} + \frac{bb}{4aa}$.
For half the Co-efficient $\frac{b}{a}$ is $\frac{b}{2a}$ the Square of which is $\frac{bb}{4aa}$.

Are you sensible of this?

Nov. Nothing appears plainer.

Phi. Since you know Something of the Nature of *Involution*, and compleating the Square, I will now give you a Notion of *Evolution* directly.

3. Of EVOLUTION.

Evolution is the Reverse of *Involution*, and shews us how to extract the Roots of any given Power.

Ex-

EXAMPLES.

xx

x Root

xxxx

xx Root.

xx bb cc

xx bbbb. dddd

x b c Root

x bb dd Root.

OBSERV. I.

When there are several Quantities in one Power, then consider which of those Powers are perfect or pure Squares of themselves; for should the first and third be so in any Power raised from a *Binominal* or *Risidual*, extract the Root of the said two Powers, and you have the square Root of the whole Quantity or Power. Thus,

$$xx + 2xb + bbb \text{ Square,}$$

$$x + b \quad \text{Root.}$$

For the Root of xx is x , and the Root of bb is b , and these two connected are $x + b$, and these I suppose the true Root; but I find it to be so upon two Trials, first $x \times b = ab$, this doubled is $2xb$ the middle Term; also $x + b \times x + b$ gives $xx + 2xb + bb$ the Power given. Again,

$$x^4 + 6xxx + 9xx \text{ Square}$$

$$xx + 3x \text{ Root.}$$

Also suppose $xxxx - 14xxbbcc + 49bbbbcccc.$

Then $xx - 7bbcc$ Root.

Here are two pure Powers, the Square of which is xx and $7bbcc$; therefore I conclude $xx - bbcc$ the Root, because the middle Member is *negative*, and the

the Square of half its Co-efficient gives 49 in the third Member.

Nov. I understand you well; but how am I to extract the square Root of Fractions?

Phi. After the same Manner. For,

OBSERV. 2.

If the first Member be a pure Power, and the Fraction also, you may consider it as a perfect Square raised from a *Binominal* or *Risidual* Root; extract therefore the Root of the Numerator for a new Numerator, and of the Denominator for a new Denominator.

EXAMPLES.

Let $xx + 3x + \frac{9}{4}$ Square
 $x + \frac{3}{2}$ Root.

For the Root of $\frac{9}{4}$ is $\frac{3}{2}$; and $x + \frac{3}{2} \times x + \frac{3}{2} = xx + 3x + \frac{9}{4}$. Again,

Let $xx + 3bx + \frac{9}{4}bb$ Square
 $x + \frac{3}{2}b$ Root. Again,

Let $xx - \frac{bx}{a} + \frac{bb}{4aa}$ Square
 $x + \frac{b}{2a}$ Root.

See *Example 6.* in *Involution.* Also,

Let $xx - dx + \frac{1}{4}dd$ Square
 $x - \frac{1}{2}d$ Root.

Do you understand it?

Tyr.

Tyr. Yes very well, except in one Thing, and that seems very odd to me.

Phi. What is that?

Nov. Why, I perceive the Root of the Fraction is larger than the Fraction itself.

Phi. Not in every Respect neither; for $\frac{1}{4}$ of dd must be more then $\frac{1}{2} d$; but I suppose you wonder that the Square Root of $\frac{1}{4}$ should be $\frac{1}{2}$, which is more than $\frac{1}{4}$ itself.

Nov. I do so.

Phi. That the Root of every simple Fraction is greater than the Square itself; you may see the Reason of this, in *Dialogue 3. Sect. 3. Note 1. and Note the 3d. Sect. 4. of the same Dialogue.*

Nov. But I wish you would demonstrate it.

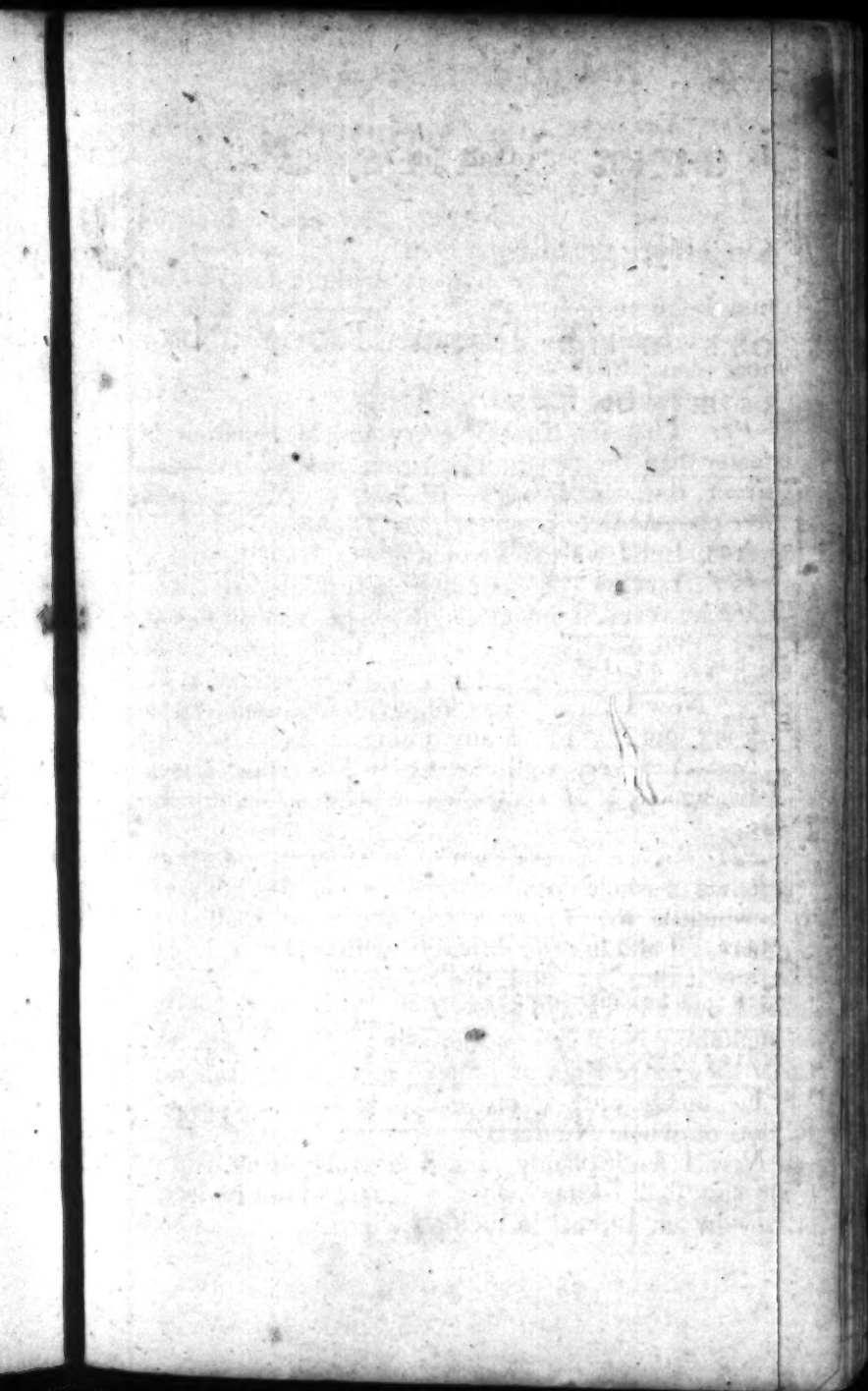
Phi. You ask Things indeed foreign to the Purpose; however, I am ready to oblige you in every Thing that may be serviceable; I shall therefore explain it by Decimal Fractions; and you will see it at once: Now I suppose you know in Decimals .25 is $\frac{1}{4}$. 5 is $\frac{1}{2}$ and .75 is $\frac{3}{4}$ of any Thing.

Nov. Yes very well, for 25 is $\frac{1}{4}$ of 100, 5 is $\frac{1}{2}$ of 10, and 75 $\frac{3}{4}$ of 100, their respective Denominators.

Phi. Right, observe then, I only set down 25 as it stands in whole Numbers, and find the Root thereof 5, which is five Times less in the Root than the Square. I also in $\frac{1}{4}$ set down 25 with a Dot or Prick before it thus .25, and the Square Root is still 5; but I put a Prick also before the .5 that being a Decimal also: Now .5 you know is $\frac{1}{2}$: By this you see that the Square Root of simple Fractions encrease in Value or Quantity in proportion to the Decrease of Roots of whole Numbers.

Nov. I see it plainly, and I heartily thank you; but how shall I know when a Square is not perfect, and how am I to act in such a Case.

Phi.



A TABLE of CONVERGING SERIES, &c.

Shewing by Inspection only,

The ROOTS and POWERS of such ROOTS to the Twelfth Root; by which any higher POWERS may be found,

Root, or First Power - - - - -	$x=1$	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=7$	$x=8$	$x=9$
Square, or Second Power - - - - -	$x^2=1$	4	9	16	25	36	49	64	81
Cube, or Third Power - - - - -	$x^3=1$	8	27	64	125	216	343	512	729
Biquadrate, or Fourth Power - - - -	$x^4=1$	16	81	256	625	1296	2401	4096	6561
Sur-solid, or Fifth Power - - - - -	$x^5=1$	32	243	1024	3125	7776	16807	32768	59049
Square-cubed, or Sixth Power - - - -	$x^6=1$	64	729	4096	15625	46656	117649	262144	531441
Second Sur-solids, or Seventh Power - -	$x^7=1$	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrate squared, or Eighth Power - -	$x^8=1$	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cube cubed, or Ninth Power - - - - -	$x^9=1$	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Sur-solid squared, or Tenth Power - - -	$x^{10}=1$	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
Third Sur-solid, or Eleventh Power - - -	$x^{11}=1$	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
Square-Cube squared, or Twelfth Power - -	$x^{12}=1$	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481

Let this fold against Page 227.

Phi. That will discover itself by the foregoing Rules; that is, if there be not two pure Squares; or if the double Rectangle under the Squares make not the middle Term: In such Cases as these, you only put this Sign ($\sqrt{}$) before it, to shew it is a surd Quantity. For this Sign is called the *Radical Sign*, or the Sign of *Irrationality*. Thus,

The square Root of xb is $\sqrt{xb}$ of $xx + dd$ is $\sqrt{xx + dd}$.

Now you see it is plain $xx + dd$ is absurd, or a Quantity that is not a perfect Square; for the Square of xx is x , and the Square of dd is d ; these connected are $x + d$; but $x + d \times x + d = xx + 2xd + dd$ consequently therefore $xx + dd$ is a surd Quantity.

So also the square Root of $xx + 2xb - bb$ is $\sqrt{xx + 2xb - bb}$, because the first is *affirmative*, and the third *negative*.

Again, The square Root of $xx + 5xb + bb$ is expressed $\sqrt{xx + 5xb + bb}$; because the middle Quantity is not just the Double of the Products of x and b .

And now, *Novitius*, I will give you a Table of the Powers, and shew you the Manner of involving them more plainly; and also more of the Nature of Investigation by Way of Exercise.

Here follows the Method of Investigation or extracting the Roots of all Powers.

THE Square Root having been spoken of before, I shall here begin with the third Power or Cube Root. And you are to take Notice, *Novitius*, that

in all the following Operations wherein e is above the second Power, that Part must be rejected.

I. Of the CUBE ROOT.

Let the given Number whose Root is to be extracted, be $= b$, and let $x + e = \sqrt[3]{b}$. Then if you involve $x + e$ to the third Power, that is, Cube it, you will have

$$\begin{array}{l|l|l}
 & 1 & x^3 + 3x^2e + 3xe^2 + xxx = b \\
 1 \div 3x & 2 & \frac{x^2}{3} + xe + e^2 = \frac{b}{3x} \\
 2 - \frac{xx}{12} & 3 & \frac{x^2}{4} + xe + e^2 = \frac{b}{3x} - \frac{xx}{12} \\
 3 w 2 & 4 & \frac{x}{2} + e = \sqrt{\frac{b}{3x} - \frac{xx}{12}} \\
 4 + \frac{x}{2} & 5 & x + e = \frac{x}{2} + \sqrt{\frac{b}{3x} - \frac{xx}{12}} = \sqrt[3]{b}
 \end{array}$$

Now this Method will always hold good in every Operation, whether you suppose $x +$ or $-$ than it really is, as plainly appears from the next Work. For,

Let $x - e = \sqrt[3]{b}$

$$\begin{array}{l|l|l}
 & 1 & x^3 - 3x^2e + 3xe^2 - e^3 = b \\
 1 \div 3x & 2 & \frac{x^2}{3} - xe + e^2 = \frac{b}{3x} \\
 2 - \frac{xx}{12} & 3 & \frac{x^2}{4} - xe + e^2 = \frac{b}{3x} - \frac{xx}{12} \\
 3 w 2 & 4 & \frac{x}{2} - e = \sqrt{\frac{b}{3x} - \frac{xx}{12}} \\
 4 + \frac{x}{2} & 5 & x - e = \frac{x}{2} + \sqrt{\frac{b}{3x} - \frac{xx}{12}} = \sqrt[3]{b}
 \end{array}$$

as before.

2 Of

2. Of the BIQUADRATE or 4th POWER.

Let	1	$x + e = \sqrt[4]{b}$
1 $\odot$ 4	2	$x^4 + 4x^3e + 6x^2e^2 = b$
$2 \div 6x^2$	3	$\frac{x^2}{6} + \frac{4xe}{6} + x^2 = \frac{b}{6x}$
$3 - \frac{xx}{18}$	4	$\frac{xx}{9} + \frac{4xe}{6} + ee = \frac{b}{6x^2} - \frac{xx}{18}$
4 $\text{as } 2$	5	$\frac{x}{3} + e = \sqrt{\frac{b}{6x} - \frac{xx}{18}}$
$5 + \frac{2x}{3}$	6	$x + e = \frac{2x}{3} + \sqrt{\frac{b}{6x^2} - \frac{xx}{18}} = \sqrt[4]{b}$

3 Of the SURSOLID or 5th POWER.

Let	1	$x + e = \sqrt[5]{b}$
1 $\odot$ 5	2	$x^5 + 5x^4e + 10x^3e^2 = b$
$2 \div 10x^3$	3	$\frac{xx}{10} + \frac{xe}{2} + ee = \frac{b}{10x^3}$
$3 - \frac{3}{80}xx$	4	$\frac{xx}{16} + \frac{xe}{2} + ee = \frac{b}{10x^3} - \frac{3}{80}xx$
4 $\text{as } 2$	5	$\frac{x}{4} + e = \sqrt{\frac{b}{10x^3} - \frac{3xx}{80}}$
$5 + \frac{3x}{4}$	6	$x + e = \frac{3x}{4} + \sqrt{\frac{b}{10x^3} - \frac{3xx}{80}}$
		$= \sqrt[5]{b}$

X

4 Of

4 Of the CUBE Squared or 6th POWER.

Let	1	$x + e = \sqrt[6]{b}$
1 6	2	$x^6 + 6x^5e + 15x^4e^2 = b$
2 15x ⁴	3	$\frac{xx}{15} + \frac{2xe}{5} + ee = \frac{b}{15x^4} - \frac{2xx}{75}$
3 - $\frac{2}{75}xx$	4	$\frac{xx}{25} + \frac{2xe}{5} + ee = \frac{b}{15x^4} - \frac{2xx}{75}$
4 2	5	$\frac{x}{5} + e = \sqrt{\frac{b}{15x^4} - \frac{2xx}{75}}$
5 + $\frac{4x}{5}$	6	$x + e = \frac{4x}{5} \sqrt{\frac{b}{15x^4} - \frac{2xx}{75}}$
		$= \sqrt[6]{b}, \&c.$

And thus may you proceed to the 7th, 8th, 9th, 10th, &c. Powers.

Now from a due Consideration, *Novissius*, of the above Work (for I cannot expect you to be perfect in it yet) you may by comparing the Roots thus investigated form Methods for finding of general Theorems to extract the Root of any higher Power without any troublesome Operation.

Nov. I shall like to know that.

Phi. Observe then, first let us compare the four last Operations, and you will find the Fractions before x increase uniformly, and that the Numerator and Denominator of each is always Unity more added to each. Thus,

$\frac{1}{2}x \frac{2}{3}x \frac{3}{4}x \frac{4}{5}x \frac{5}{6}x$, &c. &c. as follows,

$$\begin{aligned} \frac{1}{2}x + \sqrt{\frac{b}{3x} - \frac{1}{12}xx} &= \sqrt[3]{b} \\ \frac{2}{3}x + \sqrt{\frac{b}{6xx} - \frac{1}{18}xx} &= \sqrt[4]{b} \\ \frac{3}{4}x + \sqrt{\frac{b}{10x^3} - \frac{3}{80}xx} &= \sqrt[5]{b}, \&c. \end{aligned}$$

So

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So that they follow you see in a Series $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \text{ \&c.}$ from whence follows

$$\begin{array}{l} \frac{1}{2}x + \sqrt{\frac{b}{3} - xx} \\ \frac{2}{3}x + \sqrt{\frac{b}{6} - xx} \\ \frac{3}{4}x + \sqrt{\frac{b}{10} - xx} \\ \frac{4}{5}x + \sqrt{\frac{b}{15} - xx} \\ \frac{5}{6}x + \sqrt{\frac{b}{21} - xx}, \text{ \&c. \&c.} \end{array}$$

OBSERV. 2.

Again, If you compare the Power by which b is divided, you will find it increased by the continual Multiplication of x ; and that the Co-efficient of the said Power (b) is increased also by the continual Addition of 3, 4, 5, 6, 7, 8, \&c. Hence therefore evidently arises

$$\begin{array}{l} \frac{1}{2}x + \sqrt{\frac{b}{3x} - xx} \\ \frac{2}{3}x + \sqrt{\frac{b}{6x^2} - xx} \\ \frac{3}{4}x + \sqrt{\frac{b}{10x^3} - xx} \\ \frac{4}{5}x + \sqrt{\frac{b}{15x^4} - xx} \\ \frac{5}{6}x + \sqrt{\frac{b}{21x^5} - xx}, \text{ \&c. \&c.} \end{array}$$

X 2

OBSERV.

OBSERV. 3.

From what has been observed, it may easily be conceived, that the Fraction, into which xx is multiplied, are found and produced by multiplying half the Fraction annexed to x into the whole Fraction annexed to b . Whence follows,

$$\begin{aligned} \frac{x}{2} + \sqrt{\frac{b}{3x} - \frac{1}{12}xx} \\ \frac{2x}{3} + \sqrt{\frac{b}{6x^2} - \frac{1}{18}xx} \\ \frac{3x}{4} + \sqrt{\frac{b}{10x^3} - \frac{3}{80}xx} \\ \frac{4x}{5} + \sqrt{\frac{b}{15x^4} - \frac{2}{75}xx} \\ \frac{5x}{6} + \sqrt{\frac{b}{21x^5} - \frac{5}{252}xx} \\ \frac{6x}{7} + \sqrt{\frac{b}{28x^6} - \frac{3}{196}xx} \end{aligned}$$

Now in order to discover a Theorem, by which the Root of any Power may be extracted, you are to observe as follows,

NOTE 1.

That the *Denominator* of the Fraction into which x is multiplied, is always less by Unity (or 1) than the Index of the given Power, and also that the *Numerator* of the said Fraction is less by 1 than the *Denominator*.

NOTE 2.

That the Index of the Power of x , by which b is divided, is always equal to the *Numerator* of the aforesaid Fraction.

NOTE

NOTE 3.

That the Co-efficient of the said Power of x , is produced by multiplying $\frac{1}{2}$ the Index of the given Power into the said Index less by 1.

LASTLY.

The Fraction into which xx is multiplied, is the Product of $\frac{1}{2}$ the Fraction annexed to x , and the whole numeral Fraction annexed to b .

EXAMPLE.

Let the Theorem of the $\sqrt[30]{b}$ be required, then by Note 1.

$$\begin{aligned} 29 - 1 &= 28 \\ 30 - 1 &= 29 \end{aligned} \quad \text{Fraction of } x.$$

And by Note 2 follows

$$\frac{b}{x^{28}}$$

By the third Note $15 \times \frac{28}{30 - 1} = 435$, therefore $\frac{b}{435x^{28}}$. And

$$\text{By the last } \frac{14}{29} \times \frac{1}{435} = \frac{14}{12615}$$

The Theorem then when compleated is $\frac{28}{29}x +$

$$\sqrt{\frac{b}{435x^{28}} - \frac{14}{12615}xx} = \sqrt[30]{b}. \text{ Again,}$$

Let the Theorem of $\sqrt[12]{b}$ be required.

$$\text{First, } \frac{11}{12} - 1 = \frac{10}{11}x.$$

$$2dly. \frac{6}{10x}.$$

3dly. $6 \times 12 - 1 = 66$, which compleat

$$\frac{10}{11}x + \sqrt{\frac{b}{66x^{10}} - \frac{5}{720}xx} = \sqrt[12]{b}.$$

From a little Observation and Practice, *Novitius*, you may from these Examples improve yourself further in these Things. So I bid you farewell.

Nov. I beg, Sir, before you go, you would let me ask you a particular Question in Mensuration and I will trouble you no longer.

Phi. What is that pray?

Nov. Only give me your Opinion concerning the solid Content of different Pieces of Stone or Timber, whose Circumference and Length are the same. That is, suppose a Cylinder, and a regular Parallelopiped (having a Square for its Base) to be both 48 Inches Circumference and 20 Feet long, what is the Difference of their solid Content, or is there none?

Phi. Yes there is some, and a great deal too; though it is a difficult Matter to make some Persons sensible of it who pretend to understand Figures well.

Nov. Pray what is the Difference?

Phi. Very near $5 \frac{1}{2}$ Feet *Novitius*, the Content of the regular Parallelopiped being just 20 Feet, and the Cylinder 25 Feet $\frac{4}{5}$. But pray, *Novitius*, let me know the Reason of your asking this Question, for you seem to be very earnest about it?

Nov. To tell you the Truth then, Sir, there was some small Dispute between two or three of us concerning it; but I could not make them sensible there is any Difference at all.

Phi. But why did you not work them both by Figures, and that would have convinced them?

Nov. I did, and made it the same as you do; but they would not be satisfied with that, which occasioned a small Bet between us to be left to your Determination.

Phi.

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Phi. If you did it the right Way surely they could not be so ignorant ! Let me see the Method of your doing it ?

Nov. First for the square Tree, that being 48 Inches Circumference, consequently has 12 Inches upon every Side. Now 12 multiplied by 12 makes 144, the superficial Content; and this multiplied by 20 the Length, 8 divided by 144 gives just 20 Feet the Content.

Phi. Very right, and how did you proceed with the Cylinder ?

Nov. A Cylinder having a Circle for its Base (and being 48 Inches Circumference) I find first the Diameter thus, As 3. 1416 is to 1, so is the Circumference (48) to the Diameter (15. 2788 Inches). Then to find the superficial Content at the End, I multiply Half the Diameter (*viz.* 7. 6394) by Half the Circumference (*viz.* 24) and it gives 183. 3456 Inches the Area at the End. This divided by 144 gives 1. 2732 Feet; and this multiplied by 20 the Length, gives 25.464 solid Feet, which is nearly $25\frac{1}{2}$ as you observed before.

Phi. Very rightly performed; and would not this satisfy them do you say ?

Nov. No indeed; they say all the calculated Tables in Timber-Measure prove the contrary: So as I observed before it is left to you to decide.

Phi. To oblige you, *Novitius*, I will shew you (by and by) a Method that will not fail to convince them. But first I will tell you the Reason of this common Error; for you must Note, you are not the only Person that have been Witness to this Folly. As to all the set Tables they are calculated for square sided Timber only, according to Custom (for we are not to suppose every pretended Measurer a *Geometrician*) and in this the Pen and the Tables will agree

agree; and the Reason is this, they girt the Tree round, then take the fourth Part of that Circumference (vulgarly call'd the Girt by some) and multiply it by itself, then by the Length of the Tree; after which they divide it by 144, and it gives the Content in solid Feet; but then as I said before, it is only for square-sided Timber that this Method holds good. For of all other shaped Timber the Content will be more or less as I shall demonstrate hereafter, that will not fail, I believe, to convince your Friends of their Error.

If indeed the Buyer and Seller agree according to a customary way of measuring any Thing, we have no Business to meddle; but when we are called upon to do Justice between both; we must then proceed according to the just Rules of Arithmetic; which ought not in any Respect to give way to Things introduced meerly by prejudiced Ignorance, which may very well be called the Nurses of idle Custom, as you may see in a Series of Instances besides the Case before us. But to give one only,

I have heard a great many pretended Measurers affirm; that take a round Piece of Timber and let four Slabs be sawn off it, and even *then* it will contain more solid Feet than it did before. The *English* of which is, if I give you Two-pence out of a Shilling, I shall then have 14*d.* in Hand.——What Stupidity is here! Again, in a square Tree 48 Inches round, it is plain one Side is but 12 Inches; but in a round Tree that is 48 Inches Circumference, the Side of a Square equal thereto will be $13\frac{1}{2}$ Inches*, but the inscribed Square will be on each Side but about $10\frac{5}{16}$ Inches.

However, I shall leave Arithmetic and demonstrate

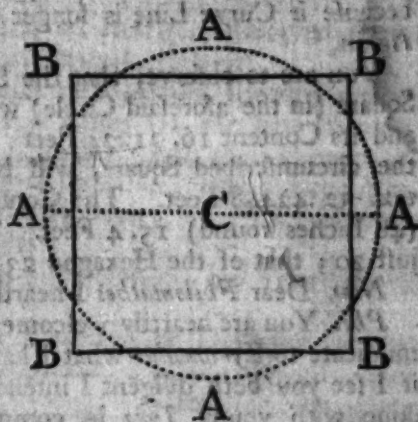
* Find the Area of the Circle, and extract the square Root thereof gives the Side of the Square equal to the Circle.

strate it by one plain geometrical Figure only, which I never knew fail to convince this sort of People; because they can see the Reason of it directly upon looking at the Scheme.

Demonstration.

As it has been proved that a Circle 48 Inches Circumference, is 15.2788 Diameter the Semi-diameter must be 7.6394. From any Scale of equal Parts therefore take off with your Compasses 7.6 Inches, and from *C* the Center, describe the Circle *A, A, A, A*, whose Circumference will then be 48. Then from the same Scale take off 12 and make this the

Side of a Square, then complete the Square *B, B, B, B*, whose Perimeter will also be 48 Inches. Now supposing this Square to be laid upon the Circle, does it not evidently appear by the Figure itself, that the *Area* or superficial Content of the Circle is



larger than the Square: For though the Square hangs over the Circle at the Points, *B, B, B, B*, yet the 4 Areas or Segments of the Circle *A, A, A, A*, are each of them larger than the former. Consequently therefore the *Area* of a Circle is larger than the *Area* of a Square, whose Perimeter is equal to the Circumference of the Circle; and if the superficial Content be greater, it is out of Dispute that the solid Content is also greater.

Nov.

Nov. This is a plain Demonstration indeed !

Phi. To be sure it is much the easiest Way : For such as are ignorant of the Square and Cube Root only think you are imposing upon them when you work such Questions at large ; but here they are convinced directly.

Nov. They are so.

Phi. From hence then it is evident, That a Circle is larger in Area than any other Figure having the same Circumference. And all Polygons are nearer the Area of the Circle according to the Number of Sides ; (as a Triangle, Square Pentagon, Hexagon, &c. &c. &c.) for the more the Sides the nearer the Circle, but they never can be quite so for this Reason, because a Curve Line is longer than a straight one. Again,

You are to observe, that the Side of the inscribed Square (in the aforesaid Circle) will be 10.8 Inches, and its Content 16.21224 Feet ; and the Content of the circumscribed Square, will be just the Double, viz. 32.42448 Feet. The Content of the Triangle (48 Inches round) 15.4 Feet. That of the Square just 20 ; that of the Hexagon 23 Feet.

Nov. Dear *Philomathes* I heartily thank you.

Phi. You are heartily welcome, only do you communicate to *Tyrunculus* what I have shewn you ; for if I see you both diligent I intend (as soon as I have done with young *Tyro* in common *Arithmetic*) to instruct you in the Rudiments of *Geography* and the Use of the *Globes*. In the mean time, *Novitius*, I bid you a hearty farewell.

Nov. Sir, I am your obliged humble Servant.

F I N I S.

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